EDDY CURRENTS : THE RESULTANT MAGNETIC FIELD WITH A PERPENDICULAR FIELD

Eddy currents are induced in a conductor when there is an incident magnetic field. These currents produce an opposing magnetic field so that the resultant field is then smaller than the incident field. But how much smaller? What is the effect of the resistivity of the conductor and its magnetic permeability? How does the resultant field vary with frequency? These questions are answered here.

1. INTRODUCTION

When an alternating magnetic field is perpendicular to a conducting plate, eddy currents are generated in the plate as shown below:

![Circulating currents and Resultant current](image)

Here the flux is represented by 25 discrete flux concentrations, and around each one is an induced current with a direction such that it produces a magnetic field which opposes the applied field. Over most of the plate these currents cancel to produce a resultant current around the edge, which opposes the applied field. An impressive video demonstration of this is given in Reference 1.

This raises a number of questions which do not seem to have been answered adequately in conventional texts:

a) What is the resultant flux, given that the induced currents oppose the incident flux?

b) What is the width of the conducting area around edge of the plate?

c) How does the resultant flux depend on the resistivity and permeability of the plate?

d) How does the resultant flux change with frequency?

This document gives a theoretical analysis supported by experiments. In outline the answers are:

a) The resultant flux is normally 50% of the incident flux for non-magnetic materials such as copper. Notice that this is far from the almost complete cancellation implied by many texts.

b) Surprisingly the resultant flux is independent of the resistivity.

c) For a thick plate the conducting width and the conducting depth are equal to the skin depth $\delta$, so that the conducting cross-sectional area is $\delta^2$. When the plate is thinner than $\delta$, the conducting
cross-sectional area is still $\delta^2$, so that the conducting width increases to $\frac{\delta^2}{t}$, where $t$ is the plate thickness.

d) The resultant flux is independent of frequency as long as the plate is large enough to allow sufficient conducting width at the lowest frequency of interest (as given by c).

Notice that this is for flux normal to the plate as shown in Figure 1.1. For flux tangential to the plate the resultant flux is very much smaller and is dependent upon the frequency and resistivity (ref 2). Attenuation can then be as high as 100 dB at high frequencies.

The theory presented here is supported by measurements of the flux through metal plates such as shown below (see Appendix 1 for details):

![Figure 1.2 Ferrite Toroid and copper sheet](image)

Two primary configurations are considered here, firstly when the flux field is smaller than the plate, and secondly when the flux field is larger than the plate.

The most significant equations are highlighted in red.

2. THE ELECTRIC FIELD AROUND A MAGNETIC FIELD

When a loop of wire is intercepted by a changing magnetic field an emf can be measured at the open terminals of the loop. This effect is well known, but what is less well known is that the emf exists whether the wire is present or not. So around every magnetic field there is an electric field, and the wire is merely a device for measuring this. So when a magnetic field is incident upon a metal plate, the electric field drives a current the magnitude of which is determined by the resistance of the conducting path.

The magnitude of the electric field is given by Lenz’s law, which states that the induced emf is equal to the rate of change of magnetic flux. So for a loop enclosing a flux $\phi$ the emf at its terminals is:

$$e = - \frac{d\phi}{dt} \text{ volts} \quad 2.1$$

The negative sign indicates that the induced emf opposes the incident flux.
For a flux which varies sinusoidally this becomes:

\[ e = - \omega \varphi \]  \hspace{1cm} 2.2

So the induced emf rises linearly with frequency, and we might then expect that the circulating current in a plate would also rise with frequency. However it is shown here that surprisingly this is not so, and the current is essentially independent of frequency. The reason is that the conducting area reduces with frequency, increasing the resistance of the path and exactly compensating for the increasing emf.

3. CONDUCTING LOOP IMMERSED IN A MAGNETIC FLUX

3.1. Flux Density along axis

Before considering flux incident on a conducting plate it is useful to consider flux \( B_0 \) incident on a single turn conducting loop:

\[ \text{Figure 3.1.1 Conducting loop in a magnetic field} \]

An emf will be generated in this loop (Equation 2.2) and if the terminals are connected together an eddy current will flow producing a flux \( B_E \) which will tend to cancel the incident flux. However the cancellation will not be complete, because a resultant flux \( B_R = B_0 - B_E \) is needed to induce an emf in the loop which will maintain the current against the loop resistance \( R \).

So the resultant flux density is

\[ B_R = B_0 - B_E \]  \hspace{1cm} 3.1.1

where

- \( B_0 \) is the incident flux density
- \( B_E \) is the flux density produced by the loop

The flux density \( B_E \) generated at the centre of the loop due to the induced current \( i \) is (Winch ref 3 p502):

\[ B_E = \mu_0 \frac{i}{2r} \]  \hspace{1cm} 3.1.2

where

- \( r \) is the radius of the loop

Notice that this is the flux density at the centre of the loop, whereas here we require the average value across the loop. This is discussed in Section 3.2 below.

There may be magnetic material near the coil with an effective permeability \( \mu_{RM} \), and so the above equation becomes:

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\[ B_E = \mu_O \mu_{RM} \frac{i}{(2r)} \]  \hspace{1cm} (3.1.3)

The emf \( e \) induced in the loop is due to the resultant flux \( \varphi_R \):

\[ e = \frac{d \varphi_R}{dt} \]  \hspace{1cm} (3.1.4)

For sinusoidal excitation, and area of loop \( \pi r^2 \) will be:

\[ e = \omega \varphi_R = \omega \pi r^2 B_R \]  \hspace{1cm} (3.1.5)

For a loop resistance \( R \), the current \( i \) induced in the loop will \( i = e/R \):

\[ i = \omega \pi r^2 \frac{B_R}{R} \]  \hspace{1cm} (3.1.6)

Substituting this into Equation 3.1.3:

\[ B_E = \mu_O \mu_{RM} \omega \pi r^2 \frac{B_R}{(2rR)} \]  \hspace{1cm} (3.1.7)

So from Equation 3.1.1

\[ B_R = B_O \cdot [\mu_O \mu_{RM} \omega \pi r \frac{B_R}{(2R)}] \]  \hspace{1cm} (3.1.8)

So

\[ \frac{B_R}{B_O} = \frac{1}{[1 + (\mu_O \mu_{RM} 2 \pi f \pi r) / (2\rho)]} \]  \hspace{1cm} (3.1.9)

The resistance \( R \) is equal to:

\[ R = \rho \ell / A_C \]

where \( \ell \) is the length of the conductor in m

\( A_C \) is its cross-sectional area of current flow (m²)

\( \rho \) is the resistivity in \( \Omega \text{m} \) (1.72 \( 10^{-8} \) for copper)

If the radius of the loop is \( r \), then \( \ell = 2\pi r \), and Equation 3.1.9 becomes:

\[ \frac{B_R}{B_O} = \frac{1}{[1 + (\mu_O \mu_{RM} \pi f \pi r) / (2\rho)]} \]  \hspace{1cm} (3.1.10)

where \( B_R \) is at the centre of the loop (see below for average across the loop).

3.2. Average Induced Flux Density

The above analysis gives the flux density at the centre of a conducting loop, but we need the average across the whole area enclosed by the conducting loop. It is shown in Appendix 4 that this is approximately equal to twice that at the centre. However there is some uncertainty in this value and so the above equation is written as:

\[ \frac{B_{RAV}}{B_O} \approx \frac{1}{[1 + (M_\varphi \mu_O \mu_{RM} \pi f A_C) / (2\rho)]} \]  \hspace{1cm} (3.2.2)

where \( M_\varphi \) is the ratio of average flux to central flux \( \approx 2 \) (see Appendix 4)
3.3. Rectangular Poles
In the above it is assumed that the poles producing the flux are circular, but alternatively they could be rectangular, and indeed it was rectangular poles used in the experiments. Appendix 2 derives the following for rectangular poles:

\[
\frac{B_{RAV}}{B_O} = \frac{1}{1 + (2 \mu_0 \mu_r \phi A_e) (a^2 + b^2)^{0.5}/ \{(a+b) \rho\}} \tag{3.3.1}
\]

4. FLUX FIELD SMALLER THAN THE CONDUCTING PLATE

4.1. Introduction
In this Section the above equation is applied to the situation where the incident flux field covers an area smaller than that of the conducting plate, as illustrated below:

The above diagram is a cross-sectional cut through the conductor where the incident flux field has been idealised in that it has constant value over the illuminated. The resultant field is the incident field minus the eddy field and this resultant field passes through the conductor.

The induced currents are shown shaded and they spread-out from the edge of the flux field due to diffusion (see Payne ref 5), both downwards into the conductor and sideways over its surface. In the above diagram this diffusion is shown as having a discrete depth and discrete width (both equal to the skin depth) but diffusion is in fact exponential so that there is some current at all depths and all widths.

It is possible that there would be current diffusion on the bottom face as well as the top face, however the equations developed here agree well with experiment and so the above seems to be correct.

It is assumed here that current flows only around the periphery of the flux because currents in the area illuminated by the flux cancel (as Figure 1.1.1). This assumption is consistent with the principle that the energy of the system will tend towards a minimum, so that the resultant flux will be minimized (this is essentially a restatement of the second law of thermodynamics). This minimum is achieved when the current encompasses as much of the flux as possible and the experiments support this.

The situation gets more complicated at low frequencies when the required conducting width can be greater than the available periphery around the flux. In this case we might expect some resultant conduction in the flux area but experiment does not support this (but see also Section 5.1).

If the flux field has an area of \(A\) the total flux will be:

\[
\Phi = B A \tag{4.1.1}
\]
where $B$ is the flux density

So the ratio of the resultant flux $\Phi_R$ to the incident flux $\Phi_O$ is

$$\frac{\Phi_R}{\Phi_O} = \left( \frac{B_R}{B_o} \right) \left( \frac{A_E}{A_O} \right) \quad 4.1.2$$

where $A_E$ is the area enclosed by the eddy current

$A_O$ is the area of the incident flux

In the case considered here where the field is smaller than the plate these two areas are the same and so $\Phi_R/\Phi_O = B_R/B_o$ and so the analysis can proceed on the basis of flux density.

4.2. Large Conductor in area and thickness

Assuming that the cross-section of the current flow is rectangular with a depth into the conductor of $d$ and conducting width $w$, then the conduction area is $A_C = d \times w$ and Equation 3.2.2 becomes:

$$\frac{B_R}{B_o} = \frac{1}{1 + \left( M_\Phi \mu_O \mu_{RM} \pi f d w \right) / (2\rho)} \quad 4.2.1$$

If the conductor is very thick the depth $d$ is equal to the skin depth $\delta$. Experiment shows that the conducting width is then also equal to $\delta$, so that the product $dw$ is equal to $\delta^2$, and this is equal to the conducting area:

$$A_C = \delta^2 \quad 4.2.2$$

The skin depth $\delta$ is given by:

$$\delta = \left[ \frac{\rho}{(\pi f \mu_O \mu_{RC})} \right]^{0.5} \quad 4.2.3$$

where $\mu_{RC}$ is the relative permeability of the conductor

Equation 3.2.2 for the circular poles then becomes:

$$\frac{B_R}{B_o} = \frac{1}{1 + \left( M_\Phi \mu_{RM} / (2\mu_{RC}) \right)} \quad 4.2.4$$

$\mu_{RC}$ is the relative permeability of the conductor

$\mu_{RM} \approx 2.9$ but dependent upon experimental set-up (see Appendix A3)

$M_\Phi \approx 2$ (see Appendix 4)

For rectangular poles Equation 3.3.1 becomes:

$$\frac{B_R}{B_o} = \frac{1}{1 + C M_\Phi \mu_{RM} / (2\mu_{RC})} \quad 4.2.5$$

where $C = (4/\pi)(a^2+b^2)^{0.5}/(a+b)$

Note that for rectangular poles with sides of ratio 2:1 (ie $a=1$ and $b=2$, as in the experiments here), then $C=0.95$ and so there is then only a 5% difference between the equation for circular poles and that for rectangular poles.

The permeability $\mu_{RM}$ is dependent upon the proximity of magnetic material to the induced currents and for the experimental set-up here its value is approximately 2.9 (see Appendix A3).

Equation 4.2.5 has been proven by the following experiment. Flux was generated in an air gap in a ferrite toroid (Appendix 1), metal plates were inserted in the gap the flux reduction was determined by the change in inductance of the wound toroid. For three plate thicknesses this was measured as follows:

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Figure 4.2.1 Flux passing through copper plates of different thicknesses

Equation 4.2.5 (in green above) predicts an inductance ratio independent of thickness and the measurements confirm this. The exception is the thin plate (0.065 mm) at the low frequencies, where more flux passed through. This is because the plate was not large enough to give the conducting width required, and this is considered in Paragraph 4.4.

Notice that Equation 4.2.5 is independent of the conductor resistivity, and this is supported by the following measurements of aluminium, lead and copper sheets:

Figure 4.2.2 Flux passing through plates of Aluminium, Lead and Copper

The flux passing through is essentially the same for all metals despite the fact that the resistivity of lead is 13 times that of copper.

The inductance ratio here was lower than that of Figure 4.2.1. The reason is not clear but the toroid here had fewer turns and a wider gap and so \( \mu_{RM} \) may have been different. Also the turns were less concentrated around the poles and so the leakage flux may have been higher.

For the usual case where the conductor has a permeability of unity and there is no high permeability material in the vicinity of the plate (ie \( \mu_{RM} = 1, \mu_{RC} = 1 \)and \( M_\phi = 2 \)), the resultant flux for circular poles is:

\[
\frac{B_R}{B_0} \approx 1/ [1+1] = 0.5
\] 4.2.7
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So as long as the plate size is sufficient to support the conducting width needed at the low frequencies, the resultant flux density is half that of the incident flux and is independent of frequency and resistivity. This implies that the resistivity could be very high (eg an insulator) and the above equation still hold but in that case the plate size would have to be extremely large.

4.3. Conductor of any size

Experiments with large sheets show that $B_R / B_O$ is to be independent of the thickness, so the product $(f A_C)$ in Equation 3.2.2 must be constant. The area must therefore reduce with $f$. Given that $A_C = w \, t$ and the thickness is constant this implies that the conducting width $w$ must be proportional to $1/f$. This is in contrast to normal current diffusion into the surface of a conductor which is proportional to $1/\sqrt{f}$ (ie skin depth).

So from Equation 4.2.2:

$$A_C = \delta^2 = t \, w'$$

4.3.1

where $w'$ is the conducting width

The conducting width $w'$ is therefore:

$$w' = \delta^2 / t$$

4.3.2

where $\delta$ is the skin depth

$t$ is the conductor thickness

So the current density in the plate must be as follows:

Notice that the conducting width $w'$ will be very large when the skin depth is large, and indeed it may exceed the dimensions of the plate. In this case the eddy current flux is reduced and $B_R / B_O$ rises and this is considered in Section 4.4 below.

(NB the electric field surrounding the flux is unlikely to extend to large distances, so the current at these distances must be due to diffusion alone.)
4.4. Conducting width limited by Plate size

When the skin depth is large the conducting width $w'$ given by Equation 4.3.2 may exceed the size of the conducting plate. The cross-sectional area for conduction $A_C$ will then be limited and the value of $B_R / B_O$ will increase, and this is seen in Figures 4.2.1 and 4.2.2 at low frequencies for the thinner plates. In this situation it is necessary to determine the effective area, and this is dependent upon the distribution in current density, and here it is assumed to vary exponentially with the distance $x$, measured from the edge of the magnetic flux field as shown in Figure 4.3.1.

So the current density can be written:

$$J_x = J_O e^{-x}$$  \hspace{1cm} (4.4.1)

The average value of this exponential is found by integrating the above equation from $x = 0$ to $x = \infty$, and this gives $J_{AV} = J_O \infty$. However experiment shows that the average value when the plate is very large is given when $x = w' = \delta^2/t$ (Equation 4.3.2) so:

$$J_{AV} / J_O = \delta^2/t$$  \hspace{1cm} (4.4.2)

If the conducting path has a width $w_1$, then the integration must be conducted from $x = 0$ to $x = w_1$. The width $w_1$ is illustrated below, where the plate is assumed rectangular of dimensions $A$ and $B$, and there is a flux field with a rectangular cross-section of dimensions $a$ and $b$. There is therefore a conducting strip around the flux periphery with a constant width $w_1$.

![Plate of size A B illuminated by flux of size ab](image)

Figure 4.4.1 Plate of size A B illuminated by flux of size ab

It is assumed that no resultant current flows in the area $ab$, illuminated by the flux. The average value of the current is then given by integrating the current density across the conducting width from zero to $w_1$:

$$J_{AV} / J_M = \int_0^{w_1} e^{-x/w'} dx$$  \hspace{1cm} (4.4.3)

$$J_{AV} / J_M = \left[ - \delta e^{-x/w'} \right]_0^{w_1} = J_m \left[ - w' e^{-w_1/w'} - (-w') \right]$$

$$J_{AV} / J_M = w' \left[ 1 - e^{-(w_1/w')} \right]$$  \hspace{1cm} (4.4.4)

The effective width is therefore reduced by the factor $[1 - e^{-(w_1/w')}]$, and so the effective width is given by:

$$w_E = w' \left[ 1 - e^{-(w_1/w')} \right]$$  \hspace{1cm} (4.4.5)

where $w_1$ is the conducting width of the plate (figure 4.4.1) $w' = \delta^2/t$
So the resultant flux density when the conducting width can be no more than $w_1$ is given by:

$$\frac{B_R}{B_O} = \frac{1}{1 + (M\phi \mu_R \mu_M \pi t w_E) / (2 \rho)}$$

4.4.6

where

$$w_E = \delta^2 / t$$

$$\delta = \left[\rho / (\pi f \mu_R \mu_M)\right]^{0.5}$$

$\mu_R$ is the relative permeability of the conductor

This equation reduces to:

$$\frac{B_R}{B_O} = \frac{1}{1 + M\phi \mu_R (1 - e^{-x}) / (2 \mu_R \rho t)}$$

4.4.7

where

$$x = w_1 t / \delta^2$$

$t$ is the plate thickness in metres

$$\delta = \left[\rho / (\pi f \mu_R \mu_M)\right]^{0.5}$$

$\mu_R$ is the relative permeability of the conductor

$\mu_R = 2.9$ in the author’s experiments (see Appendix A3)

$M\phi \approx 2$ (see Appendix 4)

$w_1$ is the width available around the flux for conduction

To evaluate this equation, measurements were made on a thin copper plate (0.065 mm thick) with $w_1$ equal to 11 mm. This gave the following (blue curve):

![Inductance ratio graph](image)

**Figure 4.4.2 Comparison of theory with measurements**

Also shown in green is the above equation with $t = 0.065$ mm, $\mu_R = 2.9$, $\mu_R = 1$ and $M\phi = 2$ and the correlation is seen to be very good.

The effective conducting width $w_E$ is shown below along with the conducting width if there had been no limitation due to plate size (ie so that $w' = \delta^2 / t$):
As expected the effective conductive width $w_E$ is asymptotic to 11 mm, the width of the plate around the flux.

5. FLUX FIELD LARGER THAN CONDUCTING PLATE

5.1. Introduction

The theory presented in the previous section assumes that when the plate is larger than the flux field, that conduction takes place only in the area of the plate surrounding the field and that there is no resultant conduction in the area illuminated by the field (as indicated in Figures 1.1 and 4.1.1). The experiments support this view.

Here we consider the situation where the flux field is larger than the conducting plate and so we might expect from the previous sections no conduction to occur, since there is no part of the plate which is not illuminated. This is clearly not the case as the following experiment shows:

![Figure 5.1.1 Inductance ratio for plate area smaller than flux area](image-url)
In the above the poles of the ferrite had nominal dimensions of 11.4 x 5.75 mm and the copper plate 8 x 5 mm with a thickness of 0.07 mm. Note that the width of the conducting strip along the long edges is equal to 5/2 = 2.5 mm.

The inductance ratio is roughly constant at frequencies above 0.3 MHz, so we can conclude that above this frequency the conducting area is reducing with frequency so that it cancels the increasing induced emf. The conducting width \( w = \delta^2/t \) is no greater than 0.2 mm in this range, so well within the 2.5 mm strip. However note that this simple model assumes that the current density is zero beyond 0.2 mm whereas there will be an exponential ‘tail’ at greater widths. Below 0.1 MHz it appears that the conducting area is not sufficient to counter the reducing emf, and significantly at 0.01 MHz \( w = \delta^2/t \) is equal to 2 mm, close to the available conducting width.

The current density therefore seems to as shown below:

![Assumed current density model](image)

*Figure 5.1.2 Assumed current density model*

Notice that not all of the current encloses all the incident flux, and this is in contrast to the situation where the plate is larger than the flux field (Figure 4.3.1), where all of the current encloses all of the flux.

### 5.2. \( \Phi_E / \Phi_0 \)

The analysis here is for a flux field which totally illuminates the conducting plate so that its area is at least the same size of the conducting plate, or larger. This is illustrated below:

![Flux field larger than conducting plate](image)

*Figure 5.2.1 Flux field larger than conducting plate*
This illustration should be compared with Figure 4.1.1, and there are two major differences: firstly, in the above, some of the flux is not intercepted by the plate and proceeds unaffected, and secondly, the current diffuses *inwards* from the edge of the plate. Not all of the current therefore encloses all of incident flux and so the induced voltage is lower and with it the eddy current and its flux.

Because the effective conducting area is smaller than that of the total plate the analysis must be for total flux and not flux density. Starting with flux density and transposing Equation 4.1.1:

\[ B_O - B_R = B_E \]  \hspace{1cm} 5.2.1

So

\[ B_E / B_O = 1 - B_R / B_O \]  \hspace{1cm} 5.2.2

Now the ratio of the eddy flux to the incident flux will be:

\[ \Phi_E / \Phi_O = (B_E A_E) / (B_O A_O) \]  \hspace{1cm} 5.2.3

where \( A_E \) is the area enclosed by the eddy current

\( A_O \) is the area intercepting the flux

Combining 5.2.2 with 5.2.3:

\[ \Phi_E / \Phi_O = \left[ A_E / A_O \right] \left[ 1 - B_R / B_O \right] \]  \hspace{1cm} 5.2.4

where \( B_R / B_O \) is given by Equations 3.2.2

\( A_O \) is the area of the conducting plate

The area enclosed by the eddy current \( A_E \) is smaller than that of the plate and is determined below

### 5.3. Rectangular Plate enclosed by Eddy Currents

Below is shown a rectangular metal plate of overall dimensions \( h \) and \( d \), and it is assumed that this is illuminated by magnetic flux having an area of \( HD \), which is larger than the plate.

![Figure 5.3.1 Conducting area](image)

At high frequencies conduction takes place around the edge of the sheet \( hd \), with a conducting width given by \( w' = \delta^2/\mu \) (Equation 4.3.2) as shown in Figure 5.1.2. The area enclosed by the eddy current \( A_E \), is assumed to be equal to the mean area of the conduction:

\[ A_E = [h - (2w'/2)] [d - (2w'/2)] \]  \hspace{1cm} 5.3.1

The ratio \( A_E / A_O \) is therefore given by:

\[ A_E / A_O = [h - w'][d - w'] / [HD] \]  \hspace{1cm} 5.3.2

where \( w' = \delta^2/\mu \)
As the frequency is reduced the area available for conduction reduces until at some frequency this area is zero. At lower frequencies the conducting width is limited to \( h/2 \) along the narrow edge and \( d/2 \) along the longer edge giving an average of approximately \( (d/2+h/2)/2 = (d+h)/4 \). The exponentially decreasing current density across the plate is therefore truncated at this distance and this can be modelled using Equation 4.4.5:

\[
w'' = w' \left[ 1 - e^{-\left( w_1/w'\right)} \right] \tag{5.3.3}
\]

where 
\[
w_1 = (d+h)/4
\]
\[
w' = \delta^2/t
\]

So overall this gives:

\[
\frac{\Phi_E}{\Phi_O} = \left[ A_E/A_O \right] \left[ 1 - B_R/B_O \right] \tag{5.3.4}
\]

where 
\[
B_{RAV}/B_s \approx 1/\left[ 1+\left( M_{\phi} \mu_0 \mu_{RM} \pi f t w'' / (2\rho) \right) \right]
\]

\[
M_{\phi} \approx 2 \quad \text{(see Appendix 4)}
\]

\[
A_E/A_O = [h-w'][d-w']/[HD]
\]

\[
w'' = w' \left[ 1 - e^{-\left( w_1/w'\right)} \right]
\]

\[
w_1 = (d+h)/4
\]

\[
w' = \delta^2/t, \quad \text{where } \delta \text{ is given by Equation 4.2.3}
\]

\[t \text{ is the plate thickness in m}\]

Applying this equation to the experiment in Section 5.1 gives:
The correlation between theory and experiment is now very good (for $\mu_{RM} = 2.9$, $\mu_{RC} = 1$ and $M_\phi = 2$)

6. CONDUCTORS WITH HIGH MAGNETIC PERMEABILITY

Equation 4.2.5 includes the relative permeability of the plate material. To test this aspect of the theory a steel feeler-gauge was inserted between the ferrite poles with the following measurements (in green):

Also shown above is the equation with the permeability $\mu_{RC}$ set arbitrarily at 250 and constant at all frequencies. It is seen that this corresponds with measurements for frequencies up to around 0.01 MHz and thereafter the flux through the plate drops to a much lower level corresponding to a permeability of around unity at 1 MHz.
It is not possible to validate the theoretical results because the permeability vs frequency for this metal is unknown, however the general trend is typical of the permeability of steels (Bowler ref 6).

7. DISCUSSION

7.1. Introduction
Good agreement has been found between the theory presented here and the experiments. However this disguises the uncertainty in the values of the permeability surrounding the induced currents ($\mu_{RM}$), and the average flux produced by the eddy currents $M_\phi$. These appear as a product in the equations ($M_\phi \mu_{RM}$) and what is certain is that good agreement with the author’s experiments is obtained when this product is equal to 5.8.
The uncertainty in the individual value of the two components is discussed below.

7.2. Effective Permeability $\mu_{RM}$
The effective permeability surrounding the eddy currents $\mu_{RM}$ was determined as 2.83 by the average of several measurements (Appendix 3). However these measurements gave a wide range of values (±50%) and so there is a large uncertainty in the final value. In addition it is not known how well these measurements actually modelled the induced current flow and the eddy flux. Best agreement with the experiments was with $\mu_{RM} = 2.9$, but this assumes that the value of the average flux density produced by the eddy currents was correct, and there is also some uncertainty in this as discussed below.

7.3. Flux Density generated by Eddy current
In calculating the average flux density produced by the eddy current it was assumed that this was equal to that at the centre of a conducting loop (Equation 3.1.2), increased by a constant $M_\phi$. The value of this constant was determined to be 2 based on the equations for the inductance of a flat loop. However, while the flat loop adequately models the induced current at low frequencies in a thin plate, it does not model the current flow at high frequencies in thick plate and so this model is incomplete and may be in error.
Appendix 1  :  EXPERIMENTAL APPARATUS

A1.1. Apparatus

To test the above theories, flux was generated by a ferrite toroid with an air gap cut into it, into which metal plates were inserted. With a coil around the ferrite the inductance was measured with and without the metal plate, and the ratio of inductance was taken as the measure of the resultant flux. Ideally all the flux generated by the coil should pass through the air gap alone. However there will be leakage inductance, and this will be especially significant when the plate is inserted since then the flux through this path will be reduced. To minimise this leakage the permeability of the ferrite should be as high as possible and Ferroxcube toroid TX25/15/10-3E5 is suitable because it has a permeability of 10,000. It has a mean magnetic path length of \( \pi (25+15)/2 \approx 60 \text{mm} \), so it has the same reluctance as an air gap of 60/10000 = 0.006 mm.

The permeability of this ferrite is shown above and it should be noted that the permeability drops rapidly above 0.5 MHz.

An air gap with a width of about 2.1 ±0.2 mm was cut into the ferrite toroid, and a 61 turn coil wound. The distribution of the turns was found to be important with best results with the turns concentrated around the poles of ferrite, and it is known that this reduces leakage flux. This is shown below:
All measurements were corrected for the self-resonance of the toroid SRF using the following equation (Welsby ref 7).

\[ L = L_m \left[ 1 - \left( \frac{f}{f_R} \right)^2 \right] \]  

Where \( L_m \) is the measured inductance \( f_R \) is the self-resonant frequency

The self-resonant frequency \( f_R \) was measured for each experimental set-up and typically was in the range 3–5 MHz depending upon the inserted conductor.

Also the lead inductance was subtracted along with the ‘one turn’ inductance which every toroid has, and these two amounted to 0.36µH.

A1.2. Flux Leakage

In the measurements it was assumed that the only flux intercepting the plate is in the air-gap between the poles. In practice there will also be some leakage flux:

a) across the diameter of the toroid

b) across the sides of the poles

To evaluate the leakage across the diameter the inductance of the toroid was measured both with and without a central screening tube as shown below:

![Figure A1.2.1 Screening tube inserted.](image)

For the above toroid (with 62 turns) measurements were made over a frequency range 0.005 to 1.5 MHz and the reduction in inductance with the tube in place was 5% on average, indicating a small leakage flux.

Appendix 2: RECTANGULAR POLES

A2.1. Resultant Flux \( B_E \)

In Section 3 the resultant flux density was determined for circular poles, and here a similar derivation is given for rectangular poles.
The resultant flux density is

\[ B_R = B_O - B_E \tag{A2.1.1} \]

where

- \( B_O \) is the incident field
- \( B_E \) is the eddy field produced by the current loop

The flux density generated by a rectangular loop at its centre is (Moulin ref 4):

\[ B_E = 2\mu_0 i (a^2 + b^2)^{0.5} / (\pi a b) \tag{A2.1.2} \]

where \( a \) and \( b \) are the sides of the rectangle

[Notice that this is the field density along the axis only, and the average density across a loop is discussed in the Section 3.2].

The emf \( e \) induced in the loop is due to the resultant flux \( \phi_R \):

\[ e = \frac{d\phi_R}{dt} \tag{A2.1.3} \]

which for sinusoidal excitation, and area of loop \( ab \) will be:

\[ e = \omega \phi_R = \omega a b B_R \tag{A2.1.4} \]

For a loop resistance \( R \), the current \( i \) induced in the loop will \( i = e/R \):

\[ i = \omega a b B_R / R \tag{A2.1.5} \]

Substituting this into Equation A2.1.2:

\[ B_E = 2\mu_0 \omega a b B_R (a^2 + b^2)^{0.5} / (R \pi a b) = 2\mu_0 \omega B_R (a^2 + b^2)^{0.5} / (R \pi) \tag{A2.1.6} \]

So from Equation A4.1

\[ B_R = B_O - 2\mu_0 \omega B_R (a^2 + b^2)^{0.5} / (R \pi) \tag{A2.1.7} \]

So

\[ B_R / B_O = 1 / [1 + 2\mu_0 \omega B_R (a^2 + b^2)^{0.5} / (R \pi)] \tag{A2.1.8} \]

The resistance \( R \) is equal to:

\[ R = \rho \ell / A_C \]

where

- \( \ell \) is the length of the conductor in metres
- \( A_C \) is its cross-sectional area of conduction in \( m^2 \)
- \( \rho \) is the resistivity in \( \Omega m \) (1.72 \( 10^{-8} \) for copper)

the length of the conducting path \( \ell = 2(a+b) \), and Equation A2.1.7 becomes:

\[ B_R / B_O = 1 / [1 + (2\mu_0 \mu_{RM} f A_c) (a^2 + b^2)^{0.5} / (a+b) \rho] \] \tag{A2.1.9}
Appendix 3 MEASUREMENT OF $\mu_{RM}$

The flux produced by the induced current is increased by the presence of the magnetic core which generates the incident flux. This is shown below, where a conducting plate is positioned between the two magnetic poles:

![Figure A3.1 Induced flux passing through the Magnetic poles.](image)

The flux produced by the induced current is increased if there is a magnetic medium in the field which it produces. Normally the area surrounding the current is air and so $\mu_{RM} = 1$. However in the experiment above, the induced field is increased by the presence of the magnetic poles.

To estimate the overall permeability we assume initially that we have a straight conductor so that the magnetic flux path is half in air and half in ferrite. The presence of the ferrite then reduces the magnetic permeability by the following ratio:

$$\text{Permeability} = \frac{1}{0.5 + 0.5/\mu_F}$$

where $\mu_F$ is the permeability of the ferrite

If $\mu_F$ is very large the average permeability around this straight wire is $\mu_{RM} = 2$.

Here the conductor surrounds the ferrite so that the effect will increase, but the value is difficult to calculate and so measurements were made, using a conducting loop to simulate the loop of induced current. The increase in inductance was then measured when this loop was placed in the position in the toroid previously taken by the metal plate. To be representative the loop was initially made rectangular with a hole cut in the middle of the same size as the cross-section of the ferrite toroid. Initially the outside dimensions were the same as the plate but this ‘loop’ had a very low inductance and too low to measure accurately.

![Figure A3.2 Measurement loops](image)
The inductive reactance was increased by increasing the number turns, even though this was no longer representative of the conducting plate. The three turn wire loop shown above and has dimensions of 13 x 5.5 mm, so slightly larger than the toroid cross-section of 10x 5 mm. It had an inductance of 151 nH (at 5 MHz) and this inductance increased by 2.9 when placed around the gap in the toroid. However similar experiments with a larger core gave an increase of 2.22 for a coil of the same diameter as the core, and 2.1 for a coil with a smaller diameter than the core. Other similar measurements were made with different loops and with a larger ferrite and gave inductance increases which averaged at 2.83. **Best agreement with experiment was given by a ratio of 2.9.**

**Appendix 4  AVERAGE INDUCED FLUX DENSITY**

Equation 3.1.10 gives the flux density at the centre of a conducting loop, whereas we require here the average across the loop. This is given by the inductance of the conducting loop, since $L = \varphi/I$, and so:

$$B_{AV} = \frac{\varphi}{A} = \frac{LI}{A} \quad A4.1$$

The flux density at the centre is (Winch ref 2) $B_C = \mu_0 i / (2r)$ and so the ratio of the average flux to that at the centre is:

$$\frac{B_{AV}}{B_C} = \frac{2L}{\pi \mu_0} \quad A4.2$$

So the inductance of the loop is required. Here the conductor is essentially a flat planar loop, and its inductance is given by Langford-Smith (ref 8 p 446) as:

$$L = R^2 N^2 / \left(8R + 11w \right) \mu H \quad \text{for } w > 0.2R \quad A4.2$$

where $R$ is the mean radius in inches

$w$ is the width in inches

Here the inner radius $r$ is fixed (by the incident flux field) and so the mean radius $R$ is given by $(r+w/2)$ and the above becomes (for $N=1$):

$$L = \frac{(r+w/2)^2}{\left[8*(r+w/2) +11w \right]} \mu H \quad A4.3$$

where $r$ is the inner radius in inches

To test this equation two loops were made as shown below:

*Figure A4.1 Measurement loops*
The dimensions were: Inner radius 27 mm (both), width 27 and 5.5 mm. Thickness 0.27 mm.

The above equation plus the measurements are shown below:

**Figure A4.2 Measured Inductance compared to Equation**

The fat loop has such a large conducting width that the measured inductance depends on the connection point and two points were used on the inner and outer of the loop. Note that the equation is not valid for \( w < 2R \). The measurements support the equation.

From Equation A4.2 the ratio of the average flux density to that at the centre \( B_{AV}/B_C \) can be calculated as follows:

**Figure A4.3 Multiplier for average flux density**

The average value of this multiplier \( M_\Phi \) is 2.
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