MEASURING THE LOSS IN VARIABLE AIR CAPACITORS

The resistance of variable air capacitors is difficult to measure because they have a very high reactance. This is tuned-out here with a transmission line inductor. Two broadcast capacitors are measured and their losses at HF are found to be mainly in their metal parts, particularly the groove shaft carrying the moveable vanes.

1. INTRODUCTION

The resistance of high quality variable capacitors is difficult to measure because of the very high series reactance. This must be tuned-out with an inductor, and this must have a very high Q if its resistance is not to dominate the measurements, and its own resistance must be calculable to a high degree of accuracy because this cannot be independently measured. Conventional helical coils cannot be used because they do not have a sufficiently high Q and, more importantly, their loss resistance cannot be calculated with sufficient accuracy. Moulin (ref 1) solved this problem by making his inductor from two parallel conductors, shorted at the far end and spaced so that proximity effect was minimal. It therefore resembled a shorted two-wire transmission line, and the capacitor to be measured was connected to the open ends (Appendix 1).

A number of authors have measured the loss of variable capacitors (refs 1, 2, 3), and found that the series resistance conformed to the following equation:

\[ R_{cap} = R_s + \alpha / (fC^2) \] 1.1

where

- \( f \) is the frequency
- \( C \) is the total capacitance

The first factor \( R_s \) is a constant resistance, due to the contact resistance of the slip rings, and the second factor is due to the loss in the insulators, where factor \( \alpha \) is a constant. The derivation of this equation is given in reference 1.

Significant equations in this report are given in red.

2. THE AUTHOR’S PREVIOUS WORK

The above equation includes the dielectric loss but does not include the loss in the conductors, and this is important at high frequencies due to skin effect. In issues 1 and 2 of this document an additional term was added to account for this metallic loss:

\[ R_{cap} = R_s + \alpha / (fC^2) + \beta (f^{0.5}) \] 2.1

Measurements of the loss were then made using the open wire line shown in Appendix 1, and values of the constants \( R_s, \alpha \) and \( \beta \) chosen to give the best agreement with measurements. However there were two main problems with this approach: firstly Equation 2.1 is not sufficiently accurate, and secondly, electromagnetic radiation from the open conductor transmission line was much higher than calculated.

These issues are addressed here, firstly with a more accurate equation.
3. MODEL OF CAPACITOR LOSS

An equivalent circuit of the variable capacitor is shown below:

![Figure 3.1 Equivalent circuit](image)

This shows two parallel paths for the current: that through the moveable vanes \((C_1)\) and that through the capacitance of the stator vanes to the body of the capacitor \((C_2)\), including via the insulators. The resistance \(R_1\) is the sum of the resistance of the sliding contact to the rotating shaft and the resistance of the metalwork of the moving vanes, including the shaft which carries them. \(R_2\) is the sum of the series resistance of the insulators plus the metallic resistance of the stator vanes. Notice that at the minimum capacitance setting the current down the \(C_2\) branch will dominate and so the series resistance tends towards the value of \(R_2\) as the capacitance is reduced.

\[
Z_T = Z_1 Z_2/(Z_1+Z_2)
\]

\[
= (R_1-j/\omega C_1) (R_2-j/\omega C_2)/ [(R_1+R_2)- j(C_1+C_2)/\omega (C_1C_2)]
\]

\[
= [R_1R_2 - R_1j/\omega C_2+ R_2j/\omega C_1 -1/\omega C_1 C_2] / [(R_1+R_2) - j(C_1+C_2)/\omega (C_1C_2)]
\] 3.1

For a high Q capacitor \(R_1R_2\) and \((R_1+R_2)\) are small compared with the other terms, and so

\[
Z_T \approx R_1 C_1/(C_1+C_2) + R_2 C_2/(C_1+C_2) - j/\omega (C_1+C_2)
\] 3.2

It is useful to express this equation in terms of the total capacitance \(C\) since this is what is measured at the terminals. Since \(C = (C_1+C_2)\) and \(C_1 = C - C_2\) then:

\[
R_{CAP} \approx R_1 (C - C_2) / C + R_2 C_2 / C
\] 3.3

where
- \(C\) is the total capacitance
- \(C_2\) is the capacitance of the fixed vanes to ground (see Section 6.3)
- \(R_1 = R_C + \alpha f^{0.5}\)
- \(R_2 = \beta + \phi f^{0.5}\)
- \(R_C\) is the resistance of the contacts

The constants in the above equation, \(C\), \(C_2\), \(R_C\), \(\alpha\), \(\beta\) and \(\phi\), must be measured. The first three can be measured with conventional test equipment and the last three with the test jig described below.
4. MEASUREMENT JIG

4.1. General
The values of resistance to be measured are very low, typically less than 0.1Ω, and conventional test equipment is then not very accurate. In the author’s previous work this problem was overcome by measuring the Q of the tuned circuit formed by the inductor and the capacitor, and using this to determine the series resistance (see Appendix 1). This is still an option but an alternative technique is given below, using a two port network analyser (ref 5):

![Two Port measurement of very low impedances](image1)

Figure 4.1.1 Two Port measurement of very low impedances

Here the two ports of the VNA are connected together and the device under test (DUT) is shunted across the connecting cable, thereby reducing the signal into the second port. For a 50 Ω system the value of the impedance $Z_{DUT}$ is given by:

$$Z_{DUT} = \frac{25 S_{21}}{1-S_{21}} \text{ ohms}$$

4.1.1

This technique gives the impedance of the DUT, and to find the resistive component the capacitive reactance has to be tuned-out with a series inductor. A practical implementation is shown below with an inductor made from copper tubing:

![Measurement apparatus with copper tube inductor (lid and ends not shown)](image2)

Figure 4.1.2 Measurement apparatus with copper tube inductor (lid and ends not shown)
A semi-rigid coaxial cable (silver-coloured) connects the two ports of the Vector Network Analyser (Array Solutions type UHF). The outer sheath of the cable has been stripped away for a short length of about 10mm, to expose the centre conductor. To this inner is soldered one end of the copper tube inductor, and its other end is soldered to the capacitor, both connections being made with short lengths of copper strip. The aluminium chassis provides electrical continuity between the capacitor body and the VNA, but to improve this connection a short copper strip is soldered between the capacitor body and the outer of the coaxial cable. To provide EM shielding the whole is contained in an aluminium U section, with a length of 500mm. The lid and ends are not shown in the above photograph.

4.2. Inductor
An ideal inductor should present the desired inductance with the minimum resistance, and this resistance should be accurately computable. Appendix 5 shows that a very good option is a single conductor transmission line.

4.3. Self-Resonant Frequency (SRF)
Measurements are made at the resonant frequency between the capacitor and the transmission-line inductor. However the inductance of a transmission-line is not constant and rises with frequency, and at a sufficiently high frequency the line itself will resonate. This is known as the self-resonant frequency (SRF) and this occurs when the length of the transmission-line is around \(\lambda/4\), and this will occur in the VHF range for the above jig. Along with this rise in inductance there is also a rise in the resistance (beyond that due to skin effect) and at the SRF the impedance of the line is many orders higher than the resistance of the conductor. This increase extends down to much lower frequencies, so that at a frequency of, say, only 1/4 of the resonant frequency the error due to this cause is 14%. So this self-resonance places a serious limitation on the maximum frequency at which accurate measurements can be made. Of course the length of the conductor could be reduced in order to raise the SRF, but then the inductance will reduce and the desired resonance with the capacitor will not be met.

The error due to SRF can be eliminated or reduced if the effect of SRF can be predicted, and Welsby (ref 7) has carried-out such an analysis and provides the following equations for the increase with frequency of the inductance and the resistance (Appendix 2):

\[
\frac{L_M}{L} = \frac{1}{1 - \left(\frac{f}{f_R}\right)^2} \tag{4.3.1}
\]

\[
\frac{R_M}{R} = \frac{1}{1 - \left(\frac{f}{f_R}\right)^2}^2 \tag{4.3.2}
\]

for \(Q > 3\).

\(L_M\) and \(R_M\) are the measured values at frequency \(f\), and \(L\) and \(R\) are the values in the absence of self-resonance and \(f_R\) is the self-resonant frequency.

Self-resonance in the above jig occurs when the total length of the conductor \(\ell\) is approximately equal to \(\lambda/4\) (see Appendix 5):

\[
\text{SRF } f_R \approx 300/ (4 \ \ell) \ \text{MHz} \tag{4.3.3}
\]

The total length includes the length of the connection leads and the length of the current path through the capacitor. The latter is difficult to assess and indeed varies with the capacitor setting, and so in practice it is better to measure the SRF. However measurement is not straightforward because at the SRF the line impedance goes to a very high value (many k\(\Omega\)) and this gives an imperceptible change in the magnitude of \(S_{21}\). However there is a measurable change in the phase of \(S_{21}\), as shown below:

4
The above plot shows the magnitude of $S_{21}$ in green and its phase in purple. The *magnitude* plot shows two resonances, the first at around 40 MHz and this is the resonance with the capacitor. The second at around 280 MHz is a transmission-line self-resonance, however this is the second resonance and the first can only be seen in the phase plot which goes through zero phase at 135 MHz. It is this first resonance which determines the rise in resistance and inductance described by the above equations. The SRF changes with the capacitor setting because the path length through the capacitor changes with setting being mainly via the moveable vanes at high capacitance settings and mainly through the capacitance to ground of the static vanes at low settings. This change can be significant and so the SRF must be measured for each capacitor setting.

### 4.4. Electromagnetic Shielding

In previous issues of this paper it was assumed that the loss due to EM radiation from the transmission line would be given by the equation for the radiation resistance of a small loop ($R_r = 31200 \times (A/\lambda^2)^2$). However this is only accurate when $A/\lambda^2$ is very small, and as the frequency is raised there is an increasing error. Since the resistance of the capacitor can be as low as 20 mΩ even a small error can be significant, and so it is essential that the circuit is shielded.

### 4.5. Capacitance v shaft angle

The value of the capacitance at each measurement frequency could have been determined from a calibration of capacitance versus shaft angle. However an alternative method was used here where the capacitance was calculated from the measurement frequency, since at this frequency it is resonant with the total inductance and this is known:

$$2\pi f_m = 1/(LC)^{0.5}$$

where $f_m$ is the measurement frequency

So

$$C = 1/[(2\pi f_m)^2 L]$$

The inductance $L$ increases with frequency due to self-resonance and so the inductance at any frequency is given by:

$$L = L_0/[1 - (f/f_{SRF})^2]$$

$L_0$ can be found since at the highest capacitance setting the capacitance is known (from low frequency measurements), and by applying Equation 4.5.1.
4.6. Overall jig resistance

The total resistance measured in the jig is that of the capacitor $R_{\text{CAP}}$ plus that of the inductor $R_W$ (increased by proximity loss $P$), and both are increased by self-resonance $S_R$:

$$R_T = (R_W + R_{\text{CAP}}) S_R$$  \hspace{1cm} (4.6.1)

$$R_W = R_{\text{dc}} (1 - e^{-x} )$$ \hspace{1cm} (ref 14)  \hspace{1cm} (4.6.2)

where  
- $x = 3.9/(d/\delta) + 7.8/(d/\delta)^2$
- $d$ is the diameter of the conductor
- $\delta$ is the skin depth $= \sqrt{\rho/(\pi f \mu)} = 66.6/f^{0.5}$ for copper
- $R_{\text{dc}} = 4 \rho \ell / \{(\pi d)^2\}$
- $\rho$ is the resistivity ($1.72 \times 10^{-8}$ for copper at $20^\circ C$)
- $\ell$ is the length in meters
- $P = 1/ \left[ 1 - (d/p)^2 \right]^{0.5}$  \hspace{1cm} (ref 14)
- $p$ is the centre to centre distance between conductors

The factor $P$ is the proximity effect and this is very small for normal spacings, adding less than 3% to the resistance for spacings of 4 times the conductor diameter or greater.

NB the above equation is for a solid conductor rather than a tube, but is sufficiently accurate as long as the thickness of the tube is greater than two skin depths ($\text{eg} > 0.13 \text{ mm at 1MHz}$).

$$R_{\text{CAP}} = R_1 (C - C_2) / C + R_2 C_2 / C$$ (see Section 3)  \hspace{1cm} (4.6.3)

where  
- $C$ is the total capacitance
- $C_2$ is the capacitance of the fixed vanes to ground
- $R_1 = R_{\text{c}} + \alpha f^{0.5}$
- $R_2 = \beta + \varphi f^{0.5}$

$$S_R = 1/ \left[ 1 - (f/f_S)^2 \right]$$ (see Section 4.3)  \hspace{1cm} (4.6.4)

$f_S$ is the self-resonant frequency

There will be a loss due to currents flowing in the jig structure but these cannot be calculated and so will be included in the capacitor losses. Also there will be some small loss from the solder joints estimated as 2 m$\Omega$ (Appendix 4).

The above does not include the resistance of the copper strips connecting the inductor. However, the capacitor always needs connection leads, and so their resistance is assumed to be part of the capacitor resistance. If this needs to be calculated equations are given in Appendix A 5.2.

To apply the above equation the total capacitance $C$ and the contact resistance $R_c$ can be measured with conventional test equipment. The factors $\alpha$, $\beta$ and $\varphi$ are optimised to give the best agreement with the jig measurements.

There is some uncertainty in the value of $C_2$, the capacitance between the fixed vanes and the body of the capacitor. If at the minimum capacitance setting the vane to vane capacitance $C_1$ was zero, then $C_2$ would be equal to the minimum capacitance. However $C_1$ cannot go to zero, and there will be small residual capacitance, and so $C_2$ will be slightly less than the minimum capacitance. In practice it was found here for Capacitor 2 that the optimum value of $C_2$ was slightly greater than the minimum capacitance and this is probably due to a small stray capacitance of around 1 pf (see also Section 6).
5. MEASUREMENTS

5.1. Introduction
Two broadcast capacitors were measured, each having two sections of equal capacitance. Capacitor 1 was rather old and well used and this is reflected in its relatively high loss. Capacitor 2 was new and unused and showed much lower loss. Their overall construction was very similar and this has a large influence on their resistance. In particular in the following photographs notice the wiping contacts, the brass shaft grooved to carry the moveable vanes, and the small ceramic insulators supporting the static vanes. Also notice the plates taking current from the fixed vanes to the terminals.

In both capacitors there was an intermittency in their resistance and this was traced to intermittent contact across their bearings. This effect is frequency dependent and is probably not detectable below 1 MHz.

5.2. Capacitor 1
Capacitor 1 is shown below. It has 2 sections each with a range of 10 pf to 380 pf measured at low frequencies. The contact resistance between the brass shaft and the body of the capacitor was measured as 1mΩ using a milli-ohm meter with 4 point contact. AC resistance measurements were made on the rear section only.

![Figure 5.2.1 Broadcast-band Capacitor 1](image)

Measurements of the overall series resistance $R_T$ (capacitor plus inductor) were made using the jig shown in Figure 4.1.2. To minimize errors a number of measurements were made at each frequency, with the capacitance changed between readings before being returned to its measurement value. It became immediately obvious that the measurements were not repeatable and there was an uncertainty in the measurements of about ±50 mΩ. At first it was thought that this was due to intermittent resistance across the wiping contacts but investigations ruled this out and showed that it was due to intermittent contact across the bearings (see Section 6).

From the measurements of the overall resistance (Equation 4.6.1) that of the inductor was subtracted to give the capacitor resistance. Below are given two curves, the first giving the maximum measured values and the second the minimum measured values. In red are the measured resistance of the capacitor and in blue are its modeled resistance:
To exaggerate the intermittent contact across the bearings the end pressure was changed, with reduced pressure for the first set of readings (Figure 5.2.2) and increased pressure for the second set of readings (Figure 5.2.3). This pressure change was achieved by adjusting the end screw and its lock nut shown in Figure 5.2.1. While this had the expected effect this is not recommended because over-tightening led to permanent damage to the capacitor. The maximum measured difference is 129 mΩ at a frequency of 50 MHz, but this large difference may have been partially due to dust between the vanes. If so then the change in end pressure may have had the effect of changing the gap between the vanes and thus the effect of the dust particles (see also Appendix 7).

The resistance reduced at high frequencies, because the capacitance was then set to a lower value and the current is then mainly through the capacitance of the static vanes to ground, rather than through the shaft carrying the moveable vanes, which has a high resistance (Section 6).

At each capacitor setting the SRF was measured and these values used in the calculations. In blue are the modeled resistance Equation 4.6.1, with $C_2$, $\alpha$, $\beta$ and $\phi$ optimised for best agreement with the measurements. The two modeled curves above have the following optimised values:

<table>
<thead>
<tr>
<th>Optimised values for maximum resistance:</th>
<th>Optimised values for minimum resistance:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_2 = 8,\text{pf}$</td>
<td>$C_2 = 6,\text{pf}$</td>
</tr>
<tr>
<td>$\alpha = 0.034$</td>
<td>$\alpha = 0.0135$</td>
</tr>
<tr>
<td>$\beta = 0$</td>
<td>$\beta = 0$</td>
</tr>
<tr>
<td>$\phi = 0$</td>
<td>$\phi = 0$</td>
</tr>
</tbody>
</table>
In both cases the factor $\varphi$ was equal to zero, and this shows that the metallic loss through the static vanes is negligibly small for this capacitor, as predicted in Appendix 8. The loss factor $\beta$ is also zero indicating that the dielectric loss was also negligible. The error bars are equal to the estimated measurement error (see Appendix 4).

5.3. Capacitor 2: Rear Section

Capacitor 2 is shown below. It has 2 sections each with a range of 9.28 pf to 343 pf, measured at 8 MHz and 1 MHz respectively. The minimum value was slightly before the maximum rotation where the capacitance increased to 9.74 pf: this becomes significant in the resistance measurements which show a rise in resistance at the maximum rotation. The contact resistance was 1.6 m$\Omega$ at DC, measured between the brass shaft and the body of the capacitor. Loss measurements were made on each of the two sections, and also of the two sections connected in parallel.

![Figure 5.3.1 Capacitor 2](image)

A number of measurements were made at each frequency as with Capacitor 1 and intermittency was seen here also although it was significantly smaller at $\pm$8 m$\Omega$ (including the repeatability of the measurements of $\pm$3 m$\Omega$, Appendix 4). From the measurements of the overall resistance that of the inductor was subtracted to give the capacitor resistance and this is plotted below. This gives the average of 6 measurements at each frequency (in red), with error bars showing the range of values measured (rather than the measurement error).
Of particular note is the rapid increase in resistance at the frequency extremes, and this occurs when the brown insulating strip supporting the moveable vanes (Figure 5.3.1) is close to the fixed vanes or in contact with them. This strip is likely to be made from phenolic resin (e.g., Paxolin) which has a dielectric constant of 5, and a very high loss tangent of up to 0.25 (i.e., \( Q = 4 \)). Also of note is the dip at 60 MHz. This has been checked several times and does not seem to be a measurement error, and the cause remains a mystery. The blue curve is the modeled resistance, Equation 3.3, and the parameters which gave the best agreement with the measurements were:

\[
\begin{align*}
C_2 &= 10.5 \text{pf} \\
\alpha &= 0.012 \\
\beta &= 0 \\
\phi &= 0
\end{align*}
\]

Notice that the metallic loss coefficient \( \alpha \) is less than the minimum values for Capacitor 1, despite being the average of the measurements for this capacitor. This suggests that the bearings here were making better and more consistent contact, confirmed by the reduced uncertainty of the resistance from ±65 mΩ for Capacitor 1 to ±8 mΩ for this capacitor. The optimum value for \( C_2 \) is slightly greater than the minimum capacitance suggesting that there was added capacitance of about 1 pf from strays. The coefficient for the dielectric loss is zero indicating that this loss is negligibly small, as is the metallic loss in the fixed vanes \( \phi \), as predicted in Appendix 8.

The \( Q \) of the capacitor is given by the following and is plotted below:

\[
Q = \frac{1}{\omega C R_C}
\]  

\[5.3.1\]
5.4. Capacitor 2: Front Section
This section of Capacitor 2 had a capacitance range from 10.5 pf (measured at 10 MHz) to 349 pf measured at 1 MHz. Capacitance at maximum rotation was 11 pf.
The resistance measurements are shown below in red below and the model in blue:

![Figure 5.4.1 Capacitor Resistance: Front Section](image1)

As with the rear section, the loss increases markedly at the highest frequency where the paxolin insulator touches the fixed vanes. However this effect is not seen at the lowest frequency, suggesting that the paxolin does not touch the fixed vanes in this section at the maximum capacitance setting.
As with the rear section, there is an unexplained dip in the resistance measurements, this time at 70 MHz.

The optimized parameters are:
\[ C_2 = 10.5 \text{ pf} \]
\[ \alpha = 0.013 \]
\[ \beta = 0 \]
\[ \phi = 0 \]

These parameters are almost identical to those of the rear section.
The Q is plotted below:

![Figure 5.4.2 Q of Front Section of Capacitor 2](image2)
5.5. Capacitor 2: Both sections in parallel
The two sections were connected in parallel with a copper strip of width 3 mm and length 18 mm (the distance between the two stator terminals). The AC resistance was measured and shown in red below, with the model in blue:

![Combined Sections](image)

*Figure 5.5.1 Capacitor 2 Resistance: Paralleled sections*

The optimized parameters are:

\[
C_2 = 19 \text{pf} \\
\alpha = 0.015 \\
\beta = 0 \\
\phi = 0
\]

The Q is plotted below:

![Q of Capacitor 2 Combined Sections](image)

*Figure 5.5.2 Q of Parallel Sections, Capacitor 2*

6. DISCUSSION

6.1. Intermittent contact
Both capacitors showed an uncertainty in the measured resistance if the capacitor shaft was turned and then reset to its initial setting. Initially this was assumed to be due to a change in the contact resistance of the
wiping contacts, however tests showed this assumption to be wrong (see Note below). The cause was the intermittent contact via the bearings between the body of the capacitor and the shaft carrying the moveable vanes: the bearings are greased to minimise wear, but this did not always prevent metal to metal contact so that sometimes there was an additional path for current from the moveable vanes in addition to that provided by the wiping contacts. Given that this additional path is in parallel with that of the wiping contacts, and these have a very low DC resistance of less than 2 mΩ, it was surprising that any change was detected. Indeed at the low frequencies for which this capacitor was designed this intermittent contact would not be detectable but at the higher frequencies used here the AC resistance of the wipers is high (10 mΩ at 50 MHz) as is that of the shaft (27 mΩ at 50 MHz). So the alternative current route through the bearings effectively halved the resistance of the shaft (see Appendix 7 for details).

[Note: To ensure the wiping contacts were clean they were sprayed with contact cleaner, the shaft turned many times and then dried with a hair dryer. This made little difference to the uncertainty. The DC resistance of the contacts when measured with a 4 point milliohm meter was very low at less than 2 mΩ. However this meter passed 80 mA through the contacts and this level of current is likely to weld the contact points together and thereby reduce the resistance. To evaluate this a current was passed through the contacts by connecting a 9v battery connected across them for about 2 seconds, and the current measured at around 2.5A. This often reduced the measured resistance but not always. The most consistent method of minimizing the contact resistance was to ‘waggle’ the capacitor shaft over an angle about ±60 degrees for several times before stopping at the final setting for the measurement.]

6.2. Parallel Sections
As expected when the two sections are connected in parallel the optimum value of $C_2$ for the combination (19 pf) is approximately equal to the sum of that of the individual sections (10.5 +10.5 =21 pf), the difference probably due to stray capacitance. The dielectric loss (β) and the metallic loss through the fixed vanes (φ) are both zero as expected from the measurements of the individual sections.

For the metallic loss we could expect that two independent capacitors of the same value in parallel would have a resistance equal to their two resistances in parallel. At 20 MHz these were measured on the individual sections as 67.5 mΩ and 58.5 mΩ, giving a parallel combination of 31 mΩ, but measured as 58.5 mΩ. This implies that there is an impedance common to both sections and calculation gives this value at 60 mΩ. The only component common to both are the spring wipers, and measurements show that their resistance is only 3 mΩ at 17 MHz (see Note below). However their inductive reactance is likely to be much higher. The wipers have a length of 25 mm but it is difficult to estimate their inductance because most of this length is close to the capacitor body, and this will reduce the inductance. Each wiper has about 0.25 mm free, and this will have an inductance of about 0.6 nH, so the four in parallel about 0.15 nH and this will have a reactance at 20 MHz of 17 mΩ. The above measurements have a large uncertainty and taking the extreme values it is just possible to explain the results. This large common impedance means that at high frequencies the two sections are not independent, and part of the current flowing in one section will flow in the other section, and this may explain why the resistance of the parallel combination is not the same as the parallel resistance of the individual sections.

[Note: Wiper resistance: a wiper from Capacitor 1 was measured and gave 29 mΩ at 50 MHz for a length of 35mm. Capacitor 2 has 4 such wipers in parallel each with a length of 25 mm, so giving a combined resistance of only 5 mΩ at 50 MHz and 3 mΩ at 17 MHz. However the wipers are in close proximity to the capacitor body and this can be expected to double the resistance to 10 mΩ at 50 MHz].

6.3. Insulator loss
The minimum capacitance of each section is around 10 pf and is the sum of the capacitance between the stator and the capacitor body plus that between the stator and the moveable vanes. In both of these the dielectric is air and so we can assume zero loss. It also includes a capacitance through the ceramic insulators, but this is very small, and has been measured as only 0.7 pf for all four insulators – so only
about 1/13th of the minimum capacitance. The loss in these insulators is therefore diluted by 13:1, so a ceramic with a Q of 500 (e.g. Steatite) will have an apparent Q of 6500 (NB the loss in a capacitor is often given as tan δ, but in RF work it is more common to use Q, and \( Q = \frac{1}{\tan \delta} \)).

6.4. Loss Budget

Bringing together the estimated losses gives the following budget at 50 MHz for each section of Capacitor 2:

<table>
<thead>
<tr>
<th>Loss Component</th>
<th>Estimated Value</th>
<th>Measured Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Insulators supporting static vanes</td>
<td>&lt; 1 mΩ</td>
<td></td>
</tr>
<tr>
<td>Shaft supporting moving vanes</td>
<td>27 (A7)</td>
<td></td>
</tr>
<tr>
<td>Plates supporting static vanes</td>
<td>3 (A8)</td>
<td></td>
</tr>
<tr>
<td>Vane resistance</td>
<td>&lt; 1 (A8)</td>
<td></td>
</tr>
<tr>
<td>Vane proximity resistance</td>
<td>&lt; 1 (A8)</td>
<td></td>
</tr>
<tr>
<td>Wiper AC resistance</td>
<td>10 Section 6.2</td>
<td></td>
</tr>
<tr>
<td>Slip ring contact resistance</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>4 solder tags</td>
<td>2 (A4.3)</td>
<td></td>
</tr>
<tr>
<td>Eddy current losses</td>
<td>15 (A9)</td>
<td></td>
</tr>
<tr>
<td><strong>Total estimated</strong></td>
<td>59 mΩ</td>
<td></td>
</tr>
<tr>
<td><strong>Measured</strong></td>
<td>67 ± 15 mΩ</td>
<td>(Capacitor 2 rear section)</td>
</tr>
</tbody>
</table>

This is a reasonable agreement and so the budget above is probably a good estimate of the losses.

6.5. Capacitor Self Inductance

The capacitor showed its own resonance when connected across the input terminal of the VNA with short leads. This resonance was dependent upon the capacitance setting: 69.7 MHz with the vanes fully meshed and 416 MHz with the capacitor at minimum capacitance.

The capacitor self-resonance is explained by series inductance: since \( f = \frac{1}{2\pi \sqrt{LC}} \) then 389 pf (measured) will resonate with 13.4 nH at 69.7 MHz. The minimum capacitance was measured as 19 pf and this resonates at 416 MHz, with 14.5 nH. These inductance values include that of the short copper connection leads which had a length of about 12 mm and diameter of 1.62 mm, and these would have an inductance of around 7.6 nH, a significant proportion of the total. So the capacitor inductance was around 6 nH.

It can be anticipated that a major inductance contribution is the central shaft carrying the moveable vanes. This has a length of 15 mm and diameter of 5 mm and so the average path length for current is 7.5 mm and the calculated inductance of this is 2 nH for a shaft without vanes (ref 9).

7. ERROR ANALYSIS

The uncertainty in the measured values of capacitor resistance is estimated as follows (Appendix 4):

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Overall Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MHz</td>
<td>±11.5 mΩ</td>
</tr>
<tr>
<td>4 MHz</td>
<td>±12 mΩ</td>
</tr>
<tr>
<td>15 MHz</td>
<td>±13 mΩ</td>
</tr>
<tr>
<td>35 MHz</td>
<td>±14 mΩ</td>
</tr>
<tr>
<td>63 MHz</td>
<td>±15 mΩ</td>
</tr>
</tbody>
</table>

8. CONCLUSIONS

A measurement procedure is given along with a suitable test jig which is able to measure the very low resistance of air variable capacitors at high frequencies. It has not been possible to calibrate the method because of the difficulty in providing a calibration standard with a sufficiently small inductive reactance.
The procedure has been used to measure the series resistance at HF of two broadcast capacitors. Their resistance has been shown to be proportional to $\sqrt{f}$ and so is mainly in the metalwork, with no significant loss in the wiping contacts or the ceramic insulators supporting the fixed vanes. In particular the shaft carrying the moveable vanes accounts for nearly half the resistance because a large part of its surface is interrupted by vanes and grooves. In contrast the plates carrying the fixed vanes have low resistance and because current can flow from each vane uninterrupted by the other vanes. The vanes themselves, both moveable and static, seem to have negligible resistance.

The resistance is lower at low capacitance settings because a smaller proportion of the current flows through the grooved shaft, and a greater proportion through the capacitance of the static vanes to ground. In the light of these findings it is probably significant that capacitors designed for high frequencies often have a current path for both the moveable vanes and the static vanes which does not go through the vanes themselves, as shown in the following example:

![Figure 8.1 High frequency capacitor](image)

Both of the capacitors measured showed an intermittency, whereby resetting the capacitor to a previous value did not always give the same resistance. This was traced to intermittent contact in the bearings and possibly dust between the vanes, and not to intermittent contact resistance in the wipers as often assumed.
Appendix 1 : PREVIOUS MEASUREMENT TECHNIQUE

In the previous issues of this paper the resistance was determined by measuring the Q (i.e. the bandwidth) at resonance and then comparing this with calculations of the Q, assuming a given loss in the capacitor. This assumed capacitor loss was then adjusted in the theoretical model to make the overall calculated Q agree with the measured Q.

The inductance coil was made from two copper pipes, each 15.03 mm outside diameter and 0.985 meter long. These were spaced at 115 mm centre to centre, and shorted at one end with a section of copper pipe. The capacitor under test was connected across the open ends with two short flexible copper strips, the whole forming a tuned circuit, as shown below.

The Q of the total circuit was measured with an AIM 4170 network analyser. This analyser could have been connected across the variable capacitor, but the resistance of the tuned circuit at resonance is very high at this point (around 60kΩ) and the analyser is not very accurate at this level. In addition the analyser will introduce stray capacitance which will upset the measurements. The analyser was therefore connected to a ‘tap’, on the coil. For this the ground terminal of the analyser was connected at a point which was assumed to be at zero potential wrt ground i.e. half-way across the end pipe forming the transmission-line short. The active port of the analyser was connected to one of the copper pipes at a distance of 150 mm from the end of the pipe, and the connection lead taken down the centre of the transmission line, forming a small loop (see figure above). This gave a resonant resistance at the analyser of around 700 Ω (at 14 MHz). The measurement of Q was not particularly sensitive to the exact dimensions of this coupling loop (≈ ± 5%), but the dimensions above gave the highest Q.

The measured Q was compared with the theoretical Q given by:

\[ Q_{\text{theory}} = \frac{\omega L}{R_s} \]

Where

- L is the measured inductance including SRF
- \( R_s \) is the total resistance (Equation 4.6.1)
The above method of determining the resistance from the measured Q is still valid, but the unscreened structure shown above is not recommended because of the high radiation loss.

**Appendix 2  PROVING WELSBY’S EQUATIONS FOR SRF EFFECTS**

All inductance coils show self-resonance and this has been attributed to self-capacitance across the ends of the coil, so that it resonates as a parallel LC circuit at a frequency known as the self-resonant-frequency (SRF). As this frequency is approached the measured resistance and the inductance increase above their low frequency value and Welsby (ref 7) gives the following equations for this increase:

\[
\frac{L_M}{L} = \frac{1}{1 - \left(\frac{f}{f_{SRF}}\right)^2} \quad \text{(A2.1)}
\]

\[
\frac{R_M}{R} = \frac{1}{1 - \left(\frac{f}{f_{SRF}}\right)^2} \quad \text{(A2.2)}
\]

$L_M$ and $R_M$ are the measured values, $f_{SRF}$ is the self-resonant frequency, and $R$ and $L$ are the low frequency values of resistance and inductance.

In fact it has been shown that a coil does not have significant self-capacitance (ref 10) and the resonance is due to the conductor acting as a transmission-line. Nevertheless the above model is surprisingly accurate (ref 10). Given that above equations are a good model of a helical transmission line (the coil) it was hoped that it was also a good model for the open wire transmission-line. This was tested with the following experiment.

The copper strip shown in Figure A9.1 was connected to the A port of the VNA, the inductance measured and normalised to the low frequency inductance to give $L_M/L_0$ and this is shown in red below:

![Figure A2.1.1 Equivalent circuit for Self Resonance](image)

*Figure A2.1.1 Equivalent circuit for Self Resonance*

The SRF was measured as 175.6 MHz and this was used in Equation A2.1 to give the blue curve above. The error was less than 3% for all frequencies up to 150 MHz (85% of the SRF).
Similar measurements for the change of resistance with frequency could not be made because the inductive reactance was greater than 30 times the resistance, and a VNA is not able to make accurate resistance measurements in the presence of such a large reactance (although Figure A 3.2 gives some confidence). However resistance measurements on inductance coils do give a close agreement with Welsby’s equations, because the coil there had a lower Q, and in addition the very good agreement for inductance gives confidence in Welsby’s equations for transmission-lines.

Appendix 3  SHIELDING

A folded conductor with one side grounded as here, is very similar to a folded monopole antenna, and at resonance this has a radiation resistance of 150 Ω. At lower frequencies this resistance will be much smaller but significant and this was confirmed by placing a conducting shielding tube around a conductor, as the following shows. The first graph shows the measured resistance without the shield, along with calculated wire resistance assuming no radiation. The second graph shows the same but with the shielding tube in place:

Figure A3.1 Measured resistance without shielding

Figure A3.2 Measured resistance with shielding
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It is seen that a major contribution to the measured resistance was that due to radiation (NB the accuracy of these measurements was poor because of the high Q. However, they are sufficient to show the improvement with the shield). The above measurements were made with the conductor connected to the A port of the VNA as shown in the photograph below. This has lower accuracy than the two port measurement system described in the main text but it allows measurements over a much broader bandwidth.

![Measurement apparatus with part of screening tube](image)

*Figure A3.3 Measurement apparatus with part of screening tube*

NB The loop used wire which had a much smaller diameter wire (0.23 mm) than shown above in order to increase its loss and lower its Q, so as to improve the accuracy of the measurements.

### Appendix 4 : ERROR ANALYSIS

#### A4.1. General
The accuracy of the technique is estimated here. Ideally this accuracy would be verified by calibration against a known standard but this is very difficult as outlined in the following section. So recourse must be made to establishing any uncertainties by theoretical means.

#### A4.2. Calibration
The jig is designed to tune out the reactance of the capacitor under test and to then measure the overall resistance. This resistance could be as low as 20 mΩ and so a test was desirable to determine the accuracy at this resistance level. The first test was to determine the noise level of the VNA and this was done with no connection between the two ports. The noise level was less than -85 dB for frequencies up to 100 MHz, and this corresponds to a shunt impedance of 1.25 mΩ when the two ports are connected together as figure 4.1. As a calibration standard, precision chip resistors are available with values from 10 mΩ upwards, in a number of lengths from 0.25 mm. This is too small for manual handling and a more convenient length would be 2mm. However the inductance of this would be around 0.7 nH (ref 11) and this will have a reactance of 220 mΩ at 50 MHz, far higher than the resistance. In principle this reactance could be tuned-out but this would require a capacitor with an extremely low ESR of much less than 1 mΩ. So testing with a low resistance is not possible at high frequencies.
A4.3. **Solder Joints**
Solder joints will have a resistance of about 0.5 mΩ. This comes from Reichenecker (ref 12) who gives measurements on the soldered joints of overlapping metal bars with an overlap area of 30 mm² (1/8” x 3/8”), and solder thickness from 0.1 mm to 1.5 mm. He found that the solder thickness made little difference and on average he measured 35 µΩ. Given that the joints here will have an area of say 2 mm², we could expect these to each have a resistance of 35x30/2= 0.5 mΩ. Given that the jig has four soldered joints these could account for about 2 mΩ.

A4.4. **Coaxial series inductance**
To expose the inner conductor of the coaxial cable the outer is cut back over a length of about 15mm to allow connection of the DUT. This exposed length will have an inductance of about 10 nH. If the DUT is connected half way along this exposed length there will be 5 nH in series with Port A and 5 nH in series with Port B.
At 50 MHz the reactance of 5 nH is about 1.5 Ω, and so the impedance of each port will increase from 50 Ω to \((50^2+1.5^2)^{0.5} = 50.02\text{Ω}\), a negligible error.

A4.5. **Uncertainty of Inductor Resistance**
The uncertainty in the inductor resistance is dependent upon its diameter, length, resistivity, permeability, temperature, proximity, width taper and SRF and these are considered below:

a) Diameter: error on the measurements here assumed to be ± 1%. Since the resistance is determined by skin effect this translates to an error in resistance of ± 1%.

b) Length: The inductor length can be measured to an accuracy of better than ± 2%.

c) Resistivity: the resistivity of plumbing copper is between 1.92 10⁻⁸ and 2.30 10⁻⁸ Ωm at 20°C (ref 15 &16). The average of these is 2.11 10⁻⁸ and this was used here, with an uncertainty of ± 9%. The AC resistance is determined here by the skin depth and this varies as the square root of the resistivity and so the uncertainty on resistance is ± 4.5%.

d) The relative permeability \(\mu_r\) of pure copper is unity, but that of plumbing copper is unknown and is assumed to be unity.

e) Temperature: the temperature of the test room was probably within ± 4°C and this gives an uncertainty in resistivity of ±0.004*4= ±1.6%.

f) The uncertainty on resistance due to the uncertainty of proximity loss is ±1%.

g) The inductor is tapered at each end to make the connections to the much narrower terminals. This taper will increase the inductor resistance and this increase is assumed to be equal to an increase in the overall length of the tube by one diameter.

Assuming that all the above errors are uncorrelated they can be added quadratically, giving an overall uncertainty in the inductor resistance of ±5.5%.

A4.6. **Error in Frequency**
The frequency error is very small and can be ignored.

A4.7. **SRF**
Experiments show that Welsby’s equation for the increase in inductance due to SRF is within ±3% of measurements. The resistance increases with \(1/[1 - (f / f_R)^2]\) rather than, \(1/[1 - (f / f_R)^2]^{0.5}\) for inductance and so the error in resistance due to this cause is ±6%.

A4.8. **Accuracy of Conductor Equation**
Equation 4.6.2 has an accuracy of ±3.6% (ref 14).

A4.9. **Repeatability**
The repeatability of the measurements has been found to be ± 3 mΩ.

A4.10. **Overall Uncertainty of Inductor Resistance**
The above errors are assumed to be uncorrelated and so can be added quadratically (ref 13), giving an overall error of ±10%.

At 50 MHz the resistance of the inductor was was 35.8 mΩ and so the uncertainty is ±3.6 mΩ. Since the resistance varies approximately as \(f^{0.5}\) the uncertainty at other frequencies is:
<table>
<thead>
<tr>
<th>f (MHz)</th>
<th>Measurement Uncertainty</th>
<th>Measurement Repeatability</th>
<th>Capacitor Intermittency</th>
<th>Overall Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MHz</td>
<td>±0.5 mΩ</td>
<td>±3 mΩ</td>
<td>±8 mΩ</td>
<td>±11.5 mΩ</td>
</tr>
<tr>
<td>4 MHz</td>
<td>±1 mΩ</td>
<td>±3 mΩ</td>
<td>±8 mΩ</td>
<td>±12 mΩ</td>
</tr>
<tr>
<td>15 MHz</td>
<td>±2 mΩ</td>
<td>±3 mΩ</td>
<td>±8 mΩ</td>
<td>±13 mΩ</td>
</tr>
<tr>
<td>35 MHz</td>
<td>±3 mΩ</td>
<td>±3 mΩ</td>
<td>±8 mΩ</td>
<td>±14 mΩ</td>
</tr>
<tr>
<td>63 MHz</td>
<td>±4 mΩ</td>
<td>±3 mΩ</td>
<td>±8 mΩ</td>
<td>±15 mΩ</td>
</tr>
</tbody>
</table>

A4.11. **Relative uncertainty**
In some of the experiments described here, the resistance of an added component was measured. This entailed measuring the resistance with and without the component and taking the difference. The error in this is that of the repeatability and assuming that the jig is unchanged except for the introduction of the component experiment shows this to be about ± 3 mΩ. If the capacitance is changed then it will increase by the capacitor uncertainty to ± 8 mΩ for Capacitor 2 far section.

Appendix 5 **TRANSMISSION-LINE INDUCTORS**

Two options have been considered for the inductor, and both are transmission-lines because the resistance can then be calculated with good accuracy. Each has advantages and disadvantages and these are discussed below.

A5.1. **Coaxial Line**
A coaxial transmission line is shown below. To form an inductor the inner and outer conductors would be shorted together at the far end.

![Coaxial cable](image)

*Figure A 5.1.1 Coaxial cable*

The major advantage of the coaxial line is that it is self-screening. However this is only true if the outer conductor can be connected to earth, and that is not possible here unless the capacitor is not connected to earth. This may be acceptable with small UHF capacitors but is not convenient for the large broadcast capacitors here.

Another disadvantage is that the inductance is relatively small unless the length is increased or the diameter of the inner is made very small. Neither is desirable because a long line will have a low SRF, and a small inner conductor will have a large resistance.
A5.2. Single Conductor Transmission-line
Below is shown a single conductor connecting the capacitor to the measurement port (the exposed inner conductor connecting the two VNA ports):

![Figure A 5.2.1 Single conductor transmission line](image)

The metal chassis forms the return conductor. The resistance of the single conductor can be calculated to a high accuracy, and if the conductor is made long enough it will have sufficient inductive reactance to tune out that of the capacitor. As the frequency is raised this inductance will increase until at a high enough frequency the line will resonate. At this high frequency the reactance of the capacitor will be low so it will form an effective short circuit to the line at this end and then this first resonance will be when the line has a length of \( \lambda/4 \). This will occur when the frequency is approximately given by the following:

\[
f_R \approx \frac{300}{(4 \ell)} \text{ MHz}
\]

where \( \ell \) is the length of the conductor. This equation is approximate because the capacitor will show a finite reactance and not a short circuit, and indeed its reactance is likely to be inductive at this high frequency so that \( f_R \) will be slightly less than given by the above equation. At this resonant frequency the input impedance will be very high.

The resistance of the conductor is given by:

\[
R_W = R_{dc} S_R / (1 - e^{-x})
\]

where \( x = \frac{3.9}{(d_w/\delta)} + \frac{7.8}{(d_w/\delta)^2} \)

\( d \) is the diameter of the conductor
\( \delta \) is the skin depth \( = \sqrt{\frac{\rho/(\pi f \mu)}{0.5}} = 66.6/f^{0.5} \) for copper

\[
R_{dc} = 4 \rho \ell / \pi d_w^2
\]

\( \rho \) is the resistivity (1.72 \( \times 10^{-8} \) for copper at 20°C)

\( \ell \) is the length in meters

\[
S_R = 1 / \left[ 1 - (f / f_R)^2 \right]^2
\]

\( S_R \) is the increase in resistance due to self-resonance (see Appendix 2).

The inductance of a straight conductor is given by:

\[
L_W = \mu_0 \ell / (2\pi) \left[ \ln (4 \ell/d) - 0.75 \right]
\]

where \( \ell \) is the length of the conductor
\( d \) is the conductor diameter

To save space the conductor can be folded back on itself. This will increase the resistance due to proximity effect given by \( P = 1 / \left[ 1 - (d_w/s)^2 \right]^{0.5} \). This is very small for normal spacings, adding less than 3% to the resistance for spacings of 4 times the conductor diameter or greater. Folding will change the inductance which is now given by:
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\[ L_W = \mu_0 \frac{\ell}{(2\pi)} [\ln (2s/d) + 0.25] \quad \text{(ref 15 p39)} \]

where
- \( \ell \) is the length of the conductor
- \( s \) is the centre to centre spacing
- \( d \) is the conductor diameter

A large conductor diameter is needed to give a low resistance, and so it is convenient to use copper tubing as shown below.

\[ R_{STRIP} = R_{sk} K_C / (1 - e^{-x}) \]

where
- \( K_C = 1 + F(0) \left[ 1.2 / e^{2.1 \ell w} + 1.2 / e^{2.1 w/t} \right] \)
- \( F(0) = (1 - e^{-0.026 \rho}) \)
- \( p = (w t)^{0.5} / (1.26 \delta) \)
- \( w \) is the strip width
- \( t \) is the strip thickness
- \( \delta \) is the skin depth = \( [\rho / (\pi f \mu)]^{0.5} \)
- \( x = 2(1+t/w) \delta/t \)
- \( R_{sk} = [\rho \ell / (w+t)] \)
- \( \rho \) is the conductor resistivity
- \( \ell \) is the conductor length

For short lengths at high frequencies the following approximation can be used for the strip resistance:

\[ R_{STRIP} \approx 1.9 \rho \ell / ((w+t) \delta) \]
As an example, for a length of 25 mm the resistance will be around 10 mΩ. Since the capacitor will always need connection leads, these can be seen as part of the capacitor if desired and then no allowance made for their resistance.

If the connection strips are close together the resistance as calculated by the above two equations will increase due proximity effect. In this case the above resistance should be multiplied by $P = 1 + 3.2/[(w/t)^{0.5} (1+2.3 \cdot g/w + 15(g/w)^2)]$ where $g$ is the distance between the strips measured from face to face, $t$ is the thickness of each conductor and $w$ is width of each conductor (ref 8). This is generally less than 1.006 if the sides of the strip conductor are spaced by more than twice the strip width, and so can then be ignored.

The problems with the above include the following:
The increase in resistance due to the changed current density at the connections cannot be calculated and so this introduces an error. Also if plumbing pipes are used the copper in these has a higher resistivity than normal copper wire, and lies between $1.92 \cdot 10^{-8}$ and $2.30 \cdot 10^{-8}$ Ωm (ref 15 &16). The average of these is $2.11 \cdot 10^{-8}$ and this was used here, but there is a potential uncertainty of ± 9%. The relative permeability $\mu_r$ of pure copper is unity, but that of plumbing copper is unknown. As a test a sample of 8 mm tube was weakly attracted to a permanent magnet indicating a permeability slightly greater than unity but its value cannot be easily determined.

Appendix 6  SHAFT RESISTANCE WITH SINGLE VANE

To test the effect of a vane on the resistance of the shaft, an 8mm copper tube was measured both with and without a vane soldered to it, as shown below.

![Figure A 6.1 Equivalent circuit at self-resonant frequency](image)

With the vane, the end-to-end resistance increased by 19 mΩ at 50 MHz, but the error is large at probably ± 50% with most of the error being due to the intermittent contact in the capacitor bearings. This experiment is a simplification of a real capacitor in that the vanes would be approximately semi-circular rather than circular, and this could be expected to reduce the effect of the vane. A theoretical determination of the resistance increase is difficult, even for the simplified configuration shown in Figure A6.1. We could imagine that the current from the copper tube flows out radially across one face of the vane, across its small thickness and then flow radially in the opposite direction down the other face. Thus the resistance of the vane would be twice that of a disc resistor having copper as its resistive medium. Harnwell (ref 4 p103) gives the resistance of a cylindrical resistor (ie a disc resistor with a central hole) as :
\[ R = \rho \frac{\ln (b/a)}{(2\pi t)} \]  

where \( \rho \) is the resistivity of the metal
\( b \) is the outside radius
\( a \) is the radius of the hole
\( t \) is the thickness of the resistive material

In this case the thickness \( t \) is equal to the skin depth, \( \delta \). For \( a = 18 \text{ mm}, b = 4 \text{ mm} \) and \( \delta = 0.01 \text{ mm} \) (50 MHz) this gives a resistance of 0.43 m\( \Omega \), or 0.86 m\( \Omega \) for the two sides. This is a factor of 44 less than measured, and so this analysis is very wrong.

Another possibility considered is that current flows on one face due to diffusion from the other. The copper disc had a thickness of 0.28 mm, equal to 28 skin depths at 50 MHz. The amplitude at one skin depth is \( 1/e \) and so at 28 skin depths the amplitude is extremely small at around \( 10^{-12} \), and so this option can be discounted.

Given that the surface current is proportional to the electric field at the surface then the experiment shows that the field across the radius of the vane must be very small, and indeed the imposed emf is between the ends of the copper tube and not across the vane as assumed in the equation above. Determining this electric field across the vane is probably the key to solving this problem but there is no technique known to the author.

Appendix 7 SHAFT RESISTANCE WITH 11 VANES

The intermittent contact suggested that the shaft resistance was significant at high frequencies, and so this aspect was explored in more detail. The capacitors show similar shaft construction with deep grooves into which the moveable vanes are inserted. The sides of the grooves are slightly wider at the surface and the vanes are pressed into these tapered grooves along with a final small crimp. In Capacitor 1 the grooves were 1.1 mm centre to centre, with a width of 0.6 mm and depth 0.8 mm, so the path length through the grooves was 2.5 times that with no grooves.

In order to measure the shaft resistance Capacitor 1 was dismantled as shown below.

![Figure A 7.1 Capacitor 1 dismantled](image)

To do this it was necessary to remove the screw and lock-nut supporting one bearing, The other bearing consisted of a small ball (not shown) between a hole in the body and a hollow in the gear end of the shaft. It was also necessary to cut away half the spring wiper strips in order to remove the shaft and vanes.

Electrical connections were made to each end of the shaft (complete with the moveable vanes) by pressure contact, and its end-to-end resistance measured in the jig similar to Figure A9.1. It was then replaced by a copper strip of the same overall length and the resistance changed by 20 m\( \Omega \) at 17.8 MHz. The strip had a calculated resistance of 12 m\( \Omega \) and so the resistance of the shaft with vanes was 32 m\( \Omega \), equal to 54 m\( \Omega \) at 50 MHz. Notice that this is the full length of the shaft, for the 2 sections, and so each section would have a measured resistance of 27 m\( \Omega \) at 50 MHz.
The following gives a theoretical analysis of this resistance. Initially considering just the grooved shaft without vanes, at high frequencies where the current flows on the surface it will be interrupted by the groves and will need to follow a longer path down the side of the groove, along the bottom and then up the other side of the groove. This will clearly increase its resistance. The bare shaft without grooves had a diameter of 6mm and half-length of 15.8 mm (from bearing to wipers) and this will have a resistance of about 3.3 mΩ (calculated at 50 MHz, and assuming brass), increasing to 8.25 mΩ if the current follows the grooves.

Consider now current in the shaft which has to traverse the vanes. The path is up one face of the vane and down the other face. To evaluate the resistance of this a single circular vane was soldered across a copper conductor, and the resistance of the pipe increased by about 19 mΩ at 50 MHz (Appendix 6). Since there are 11 vanes, and each is a semi-circle (approximately) then this route for current will have a resistance of about 400 mΩ, much larger than that calculated for the grooved shaft alone, and indicating that the part of the circumference of the grooved shaft not carrying vanes is the dominant path for the current. So current from each moveable vane flows down the un-vaned part of the shaft, rather than through the other vanes.

Notice that the vanes occupy about half of the circumference of the shaft, with an extension tab on each vane providing mechanical support over a further ¼ circumference, so that only about ¼ circumference is exposed grooved shaft. However closer inspection shows the vane extensions protrude beyond the shaft surface to provide raised peaks. It is assumed that these have the same effect as the grooves in the shaft and so the vane extensions can be ignored if it is assumed that the vanes occupy about half the circumference. The resistance of the half-length shaft with vanes will increase to 16.5 mΩ (2 x 8.25) at 50 MHz. This is less than that measured above (27 mΩ), and so we can assume that the path of the current was more constrained than assumed above and/or the effect of the grooves is greater than merely the increase in path length.

When the bearing makes contact we can expect half the vane current to now flow through the bearing and so the effective resistance of the shaft will halve, to give a variation of 13.5 mΩ (±7 mΩ), and this is close to that seen in the measurements of Capacitor 2 of ±8 mΩ.

However for Capacitor 1 the variation was much larger at ±60 mΩ. So the shaft resistance was probably not the only cause of the resistance uncertainty in this capacitor, and a major contribution could have been dust between the plates. Field & Sinclair (ref 3) show the resistance increasing by a factor of up to 3 when the capacitor was dusty, and indeed Capacitor 1 was well used and had not been cleaned.

Appendix 8  THE RESISTANCE OF THE VANES

A8.1.  DC Resistance of static vanes

The DC resistance of the static vanes of Capacitor 1, terminal to terminal, was measured as 1.48 mΩ using a four port milliohm meter with a quoted accuracy of ±1.5%. The calculated value was 0.008 mΩ for the 10 vanes in parallel but not including the resistance of the supporting plate and terminal. This was therefore a factor of 185 smaller than measured. However the resistance of a single vane was measured as 0.08 mΩ exactly as calculated (to measure this one end vane was bent away from the others so that meter clips could be connected directly to the vane). This suggests that the main resistance component was between the terminal post and the vanes, and this consists of a large solder joint and the brass (?) supporting plate. Confirmation was provided by measuring the DC resistance between the terminal and the plate and this gave 0.48 mΩ, and the two ends then give a major fraction of the measured terminal to terminal resistance. In trying to explain this resistance theoretically the plate would have to have a very high resistivity, and so assuming bronze with \( \rho = 2.5 \times 10^7 \) (14 times that of copper) but this would still only give 0.1 mΩ. The solder joint would have a similar resistivity and so increase this to say 0.2 mΩ, still less than half that measured. The remaining resistance could be in the crimp connection between the vanes and the plate.

So in summary the DC measurements show that the static vanes themselves have a very low resistance and the major resistance is that of the vane supporting structure at each end consisting of a supporting plate and the solder joint. However explaining this high resistance is difficult unless the resistivity of the plate is assumed to be very poor.
A8.2. **AC Resistance of Static Vanes**

The static vanes have terminals at each side of the capacitor, so measuring the resistance of these vanes is very convenient. However their resistance at high frequency is increased considerably by proximity to the capacitor body (Appendix 9). In principle the measurements could be corrected for this but the increase is so large ($>10$) that there would be considerable error. Given this measurement problem, a theoretical estimate is given below.

From the DC measurements above it can be assumed that the major AC resistance is in the connection plates and that the resistance of the vanes themselves is negligible. The static vanes are connected together by plates, one on each side of the capacitor, and each vane is crimped into these with two small tags on each side. The current therefore spreads out radially from these tags, and we can make a crude assumption that there is one tag and that the static plates are semi-circular (only an approximate analysis is needed here because the vane resistance turns-out to be negligibly small). The resistance then of each face will be twice that of a disc resistor (see Harnwell ref 4, p103) of thickness equal to one skin depth:

$$R = 2 \rho \ln (b/a)/(2\pi \delta)$$  \hspace{1cm} A8.4.1

where \( \rho \) is the resistivity of the metal
\( b \) is the outside radius
\( a \) is the radius of the hole
\( \delta \) is the skin depth

The hole in this case is the tag and this has a size of about 1mm. The vane material is aluminium so $\rho = 2.65 \times 10^{-8}$. For $a \approx 7 \text{ mm}$, $b=1\text{ mm}$ and $\delta=0.02 \text{ mm}$ (19.6 MHz) this gives a resistance of 0.8 mΩ. There is a similar radial current leading into the opposite terminal and so the resistance is doubled to 1.6 mΩ. There are 10 static vanes, each with two sides so the combined parallel resistance is 1.6/20 = 0.08 mΩ.

As with the moveable vanes connecting them together will add resistance. However in contrast to the moveable vanes the connection here is via conducting plates (which seem to be brass), and in these plates the current from each vane can flow without having to traverse the other vanes. So we can expect the resistance will be very much lower than that of the grooved brass rod carrying the moving vanes. However the DC measurements suggest that these plates have a high resistivity of at least $\rho = 2.5 \times 10^{-7}$, and assuming the current spreads radially from the solder joint the above equation gives a resistance of 3 mΩ at 50 MHz, assuming current flows on both faces.

A8.3. **Experiment 2: Vane proximity loss**

The resistance of the static vanes will be increased by the proximity of the moveable vanes, and vice-versa. To assess this, the resistance of the static vanes of Capacitor 2 was measured both with meshing by the moveable vanes and with no full meshing. This was done utilizing the fact that the static vanes have a terminal on each side, and so could be connected in series with the inductor. The moveable vanes are of course connected to the body of the capacitor and this has a large surface area and so has significant capacitance to the chassis. To minimise the changing effect of this as the capacitance C is varied the body was connected to one of the static terminals.

The resistance at 17.7 MHz was measured at maximum meshing and minimum meshing, with 6 measurements at each setting and averaged. The resistance showed no significant change with meshing at less than 1 mΩ (but the measurement uncertainty was probably ± 5 mΩ). One potential complication in this experiment was that the resistance could be expected to reduce when fully meshed because the resistance of the moveable vanes is then in parallel with that of the static vanes via the series capacitive reactance between them. However this will have a negligible effect because the capacitive reactance at 17.7 MHz is so high at 25Ω compared with the vane resistance of $>1$ mΩ.

Previous work with parallel conductors shows that we can expect the resistance with proximity would be up to twice that with no proximity (ref 8), so adding 0.08 mΩ to the static vanes. The measured value of < 1 mΩ is consistent with this.
Appendix 9  EDDY CURRENT LOSSES

Currents flowing in the capacitor body will produce loss, and these currents will include those induced by nearby conductors, in particular the vanes. To test this a vane was simulated with a copper sheet and the capacitor body simulated by a galvanised steel bracket:

![Image of measurement setup](image)

*Figure A 9.1 Measurement of induced loss*

Note: the inductor here was made from copper tube, rather than the copper strip shown here. The resistance was then measured both without the steel bracket and then when it was brought close to the copper sheet. For each measurement condition at least six measurements were made, the average taken and the two averages subtracted, to give the following:

![Table showing measured proximity resistance](table)

*Figure A 9.2 Measured proximity loss for galvanized steel plate*

The gap between the end vane and the steel body of the capacitor was about 3 mm and so the proximity loss will be 6.3 mΩ at 19.6 MHz. Assuming R proportional to $f^{0.5}$ then at 50 MHz this gives 10 mΩ. This will apply to the two end vanes since these are adjacent to the body. Given that there are 10 vanes then the effect will be diluted 2/10 so the overall resistance due to this effect will be 2 mΩ at 50 MHz. In addition all the vanes will be partially affected because of the proximity of the base of the capacitor body to their thickness. To test this a steel plate was brought close to the exposed edges of the moveable vanes and the resistance increased by around 13 mΩ giving a total proximity loss of 15 mΩ at 50 MHz.
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Enquiries to paynealpayne@aol.com