THE PROXIMITY LOSS IN RECTANGULAR CONDUCTORS WITH OPPOSING CURRENTS

There are significant theoretical problems in analysing the proximity loss in rectangular conductors and no closed form equations exist. Here the proximity loss was measured for a range of rectangular conductors and a semi-empirical equation was produced which matched those measurements.

1. INTRODUCTION

A DC current flowing in a conductor is distributed uniformly across its cross-section, but at high frequencies the current concentrates in a thin skin around the periphery (ref 1). In addition with a rectangular conductor there is a higher current density at the corners, as indicated below :

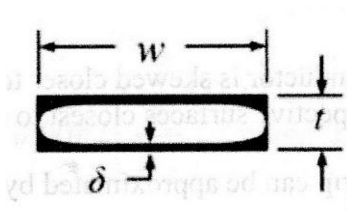


Figure 1.1 Current density at high frequencies

When a second conductor with an opposing current is brought close to the first the current now concentrates on the inner faces at the expense of the outer faces :

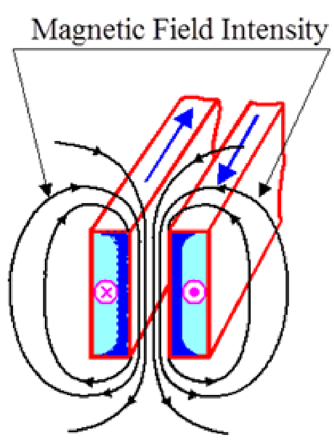


Figure 1.2 Magnetic field and current density in parallel conductors

This redistribution of current is such as to minimise the inductance of the loop formed by the go and return currents, since this minimises the stored energy ($E= LI^2$). This is in accordance with the principle of minimal energy which is a consequence of the second law of thermodynamics. Another way of looking at this is that any current flowing on the outside faces of the conductors will be presented with a greater inductive reactance than currents flowing on the inside faces.

The concentration of current on the inner faces reduces the area carrying current and so the resistance increases, and this *increase* is known as the proximity loss. This is difficult to calculate, but is better understood in circular conductors because the magnetic field is symmetrical about their axes. However even here assumptions must be made and the resultant equations are not exact (ref 1). The rectangular conductor is even more difficult to analyse and no closed form equation exists, not even an approximation. Here measurements are made and empirical equations developed which give a good agreement with the measured values.

There are two primary ways in which two rectangular conductors can be configured : width to width or thickness to thickness, and here designated as Configuration1 and Configuration 2 :

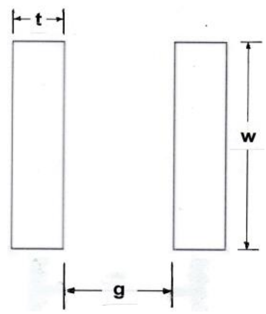


Figure 1.3 Configuration1 : width to width

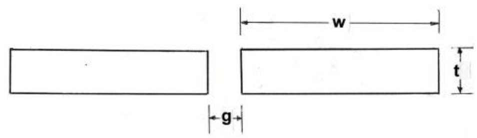


Figure 1.4 Configuration 2 : thickness to thickness

Experiment shows that the proximity loss increases with frequency and at an ever reducing rate until a limiting value is reached. So the experimental task here was to determine this limiting value and the change in loss with frequency.
Firstly we consider the resistance of a single isolated conductor.

2. RESISTANCE OF A SINGLE ISOLATED CONDUCTOR

The AC resistance of a rectangular conductor with no proximity loss has been analysed by the author (ref 2), and the equation is given below :

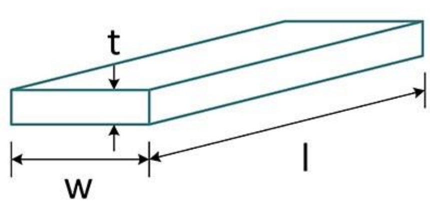


Figure 2.1 Single isolated conductor

$$\mathbf{R_{AC} / R_{DC} \approx [\, K_C / (1- e^{-x}) \,]} \qquad \mathbf{2.1}$$

where $K_C = 1 + F_{(f)}\,[1.2/ e^{2.1\, t/w} + 1.2/ e^{2.1\, w/t}]$

$F_{(f)} = (1- e^{-0.026\, p})$

$p = A^{0.5}/ (1.26\, \delta)$

A is the cross-sectional area of the conductor

δ is the skin depth = $[\rho/(\pi f \mu)]^{0.5}$

$x = [2\, \delta/t\, (1+t/w) + 8\, (\delta/t)^3 /(w/t)] / [(w/t)^{0.33}\, e^{-3.5\, t/\delta} +1]$

w is the width of the conductor in metres

t is its thickness in metres

f is the frequency in Hz

The dc resistance is equal to $R_{dc} = \rho\, \ell / (wt)$, where ρ is the material resistivity in Ωm (= $1.72\ 10^{-8}$ for copper) and ℓ is the length of the conductor in metres

NB the factor $1.2/ e^{2.1\, w/t}$ is extremely small for w/t above 3, and indeed at w/t above about 300 Excel is unable to calculate the value and returns an error message.

3. LIMITING VALUE OF PROXIMITY LOSS

Experiment shows that the proximity loss increases with frequency towards a limiting value. This raises the question as to how high the measurement frequency must be to determine this limiting value? Initially it was thought that it would be sufficient if the frequency was high enough for the skin depth to be small compared with the smallest dimension of the conductor, say 10% of the conductor thickness. However this is not so and the loss continues to rise as the frequency is raised, albeit at a reducing rate as shown by the following curve

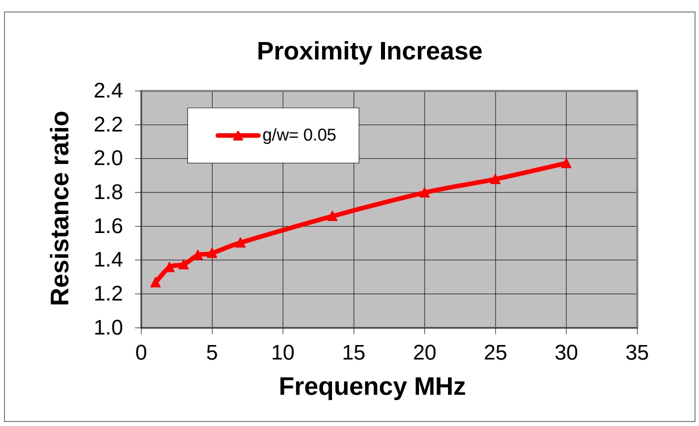


Figure 3.1 Increasing proximity loss with frequency

This curve shows the ratio of the resistance of the conductor with proximity loss to that of the conductor with no proximity loss, as measured for w/t =1 and g/w = 0.05.
The skin depth was 10% of the thickness at 7 MHz, and yet the proximity loss continued to rise at higher frequencies. The rise was found to be exponential (see below) and so to measure at 90% of the limiting value would require measurements at a frequency of over 500 MHz. While this would be possible in principle there are serious practical difficulties – see note 1 below.

It became clear that the frequency should be sufficiently high for Equation 2.1 to be close to *its* asymptotic value and the critical parameter here is $F_{(f)}$ which increases at an exponentially decreasing rate with frequency. Experiment showed that the proximity loss followed the same curve as $F_{(f)}$, as shown below :

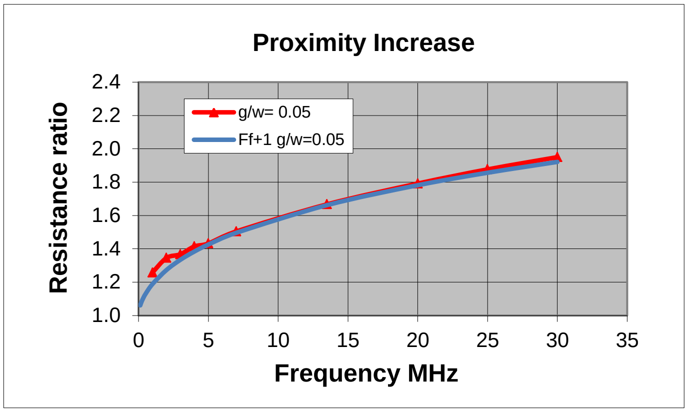


Figure 3.2 Measured proximity loss with (1+k $F_{(f)}$)

The blue curve is of the equation

$$R/R_O = (1+ k F_{(f)}) = 1+ k (1- e^{-0.026 p}) \qquad 3.1$$

where R_O is the resistance of the isolated conductor

The factor k has been adjusted in the graph to give the best agreement with the measured curve, and this was achieved at k = 2.91 in this case. The limiting value of Equation 3.1 (the maximum value) was therefore R_{LIM} / R_O = 2.91+1 = 3.91 and this then is the limiting value of the proximity loss for this w/t of unity and spacing g/w of 0.05.

This was repeated for measurements with other spacings and for other w/t ratios, to find their respective values of R_{LIM} / R_O, with the fit to the above equation being determined by the least squares error.

It is important to emphasise that the limiting value has been found by *extrapolating* the measured values, and on the assumption that the variation with frequency is given by Equation 3.1.

[Note 1: The SRF of the folded conductor would have to be around 2.4 GHz if the effects of self-resonance are to be less than say 10% (see Appendix 3), and this would entail a folded length of no more than 37 mm, and then end effect could become significant, unless the conductors are very small. This in turn would require very small gaps and physically the whole experiment would become extremely difficult. Also the resistance at 500 MHz would be below 0.5Ω and measurement error would be very large. This problem is explained in more detail in reference 2].

4. MEASUREMENTS AND EMPIRICAL EQUATION : CONFIGURATION 1

4.1. Introduction

In this Section the limiting value is determined for Configuration 1. Measurements were made of the resistance increase due to proximity of conductors with w/t ratios of 1, 2.6, 10.9 and 15.3. The conductor to be measured had a length of between 0.3 and 0.6 m and was folded back on itself and a spacer placed between the two halves.

Details of the measurement technique and the calibration of the equipment are given in the Appendices 1&2.

4.2. Calibration with loop

The overall objective was to measure the *increase* in resistance due to proximity, and so the first measurement with any conductor was to measure its resistance with zero proximity loss. Strictly this cannot be achieved in practice because the two ends of the conductor have to be brought together for connection to the test equipment. The lowest proximity loss is then with the conductor formed into a circular loop and it can be shown that the proximity loss is then the same per unit length as that for two infinite straight conductors placed parallel at a distance D apart, where D is the diameter of the loop (Moullin ref 3 p 348). For the conductors here the proximity loss with such a loop was quite negligible (< 0.1%), and so it was advantageous to fold the conductors back on themselves but with a wide spacing, since this gave a higher self-resonant frequency (SRF) than the circular loop allowing measurements to a higher frequency (see Appendix 3). The folded spacing in this case was around 15mm and this gave a proximity loss of much less than 1%, as calculated from the equation eventually developed from the measurements. For ease of description this folded conductor with wide spacing is called the calibration 'loop'.

4.3. Summary of Measurements

The limiting value of the resistance for various w/t ratios and various conductor spacings was extrapolated from the measurements as described in Section 3, and these values are plotted as red dots in the graphs below. Also shown in the graphs is the following empirical equation :

$$R_{LIM} / R_O = 1+ 3.2/ [(w/t)^{0.5} (1+2.3\ g/w + 15(g/w)^2] \qquad 4.3.1$$

where R_{LIM} is the limiting value of the resistance with proximity loss
R_O is the resistance of the isolated conductor
g is the distance between conductors measured from face to face
t is the thickness of each conductor
w is width of each conductor

Notice that the factors in brackets give respectively the dependency on w/t and the dependence on g/w. As expected the proximity loss reduces with increasing g, but not expected was that the proximity loss reduces for wide thin conductors.

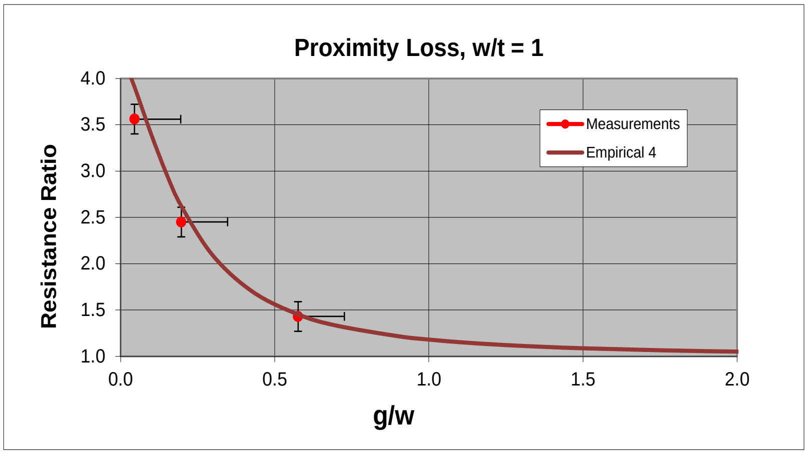


Figure 4.3.1 Limit of proximity loss for w/t = 1

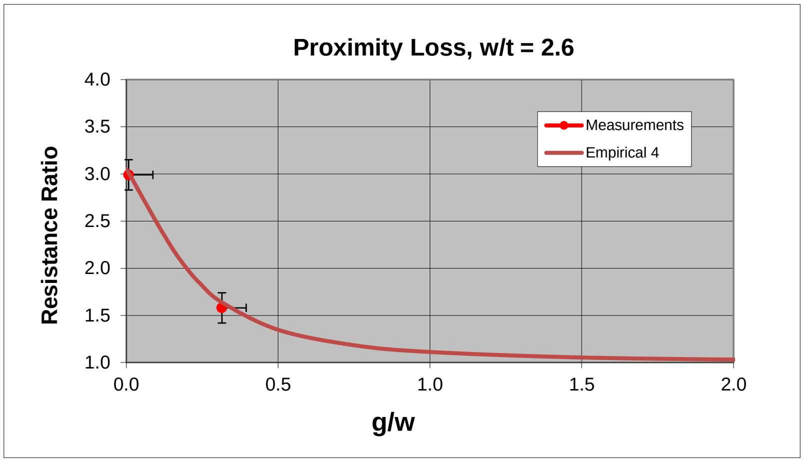


Figure 4.3.2 Limit of proximity loss for w/t = 2.6

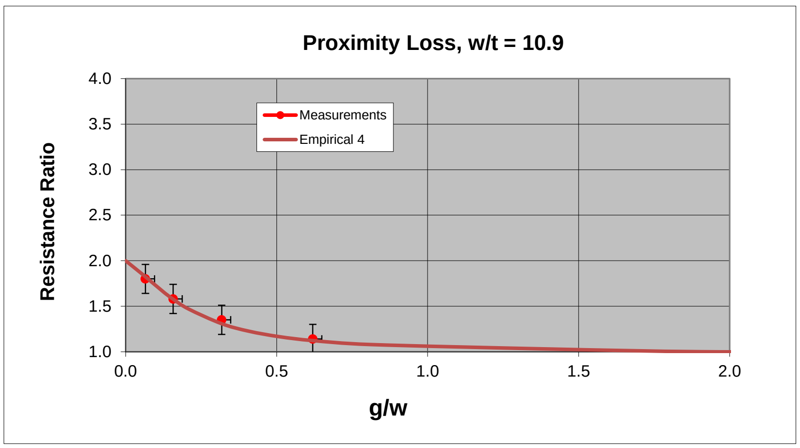


Figure 4.3.3 Limit of proximity loss for w/t = 10

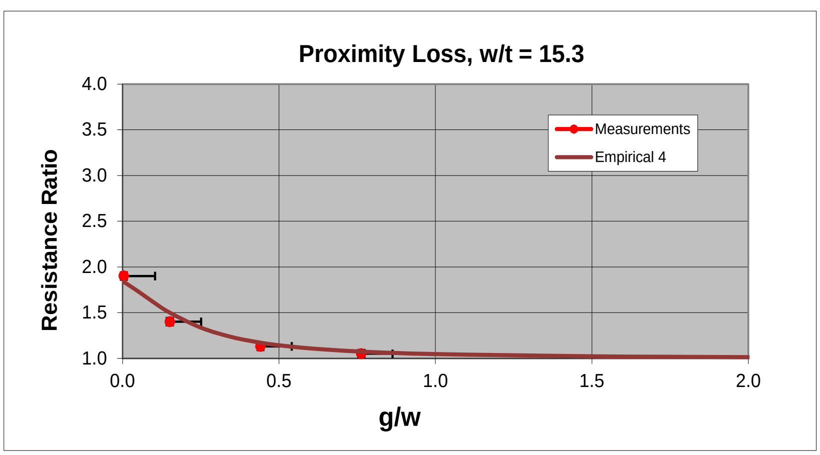


Figure 4.3.4 Limit of proximity loss for w/t = 15.3

The values of the error bars in the above graphs are determined in Appendix 2.

Notice that in each of the graphs above the x and y axis have the same maximum values of R_{AS} /R_O =4 and g/w =2, and this then emphasises that the proximity loss reduces with increasing w/t.

In each case the x axis is the ratio of the gap to the strip width g/w (Figure 4.3.4) rather than the more common s/t, where s is the centre to centre spacing and t is the thickness. This was done because it was discovered that the simple equation above then gave a reasonable correlation with all the measurements but such a simple equation could not be found when the variable was s/t.

[NB another empirical equation which matches the measurements is :

$$R_{LIM} / R_O = 1+ 3.2/ [(w/t)^{0.5} (1+g/w)^{4.2}] \qquad 4.3.2$$

5. MEASUREMENTS AND EMPIRICAL EQUATION : CONFIGURATION 2

In this Section the limiting value is derived for Configuration 2.
The square conductor (w/t =1) is common to both Configuration 1 and 2. If its thickness in Configuration 1 is increased slightly (so that w/t <1) it would be surprising if Equation 4.3.1 did not still hold, but crucially it then becomes Configuration 2. So from this argument it can be assumed that Equation 4.3.1 will still apply for Configuration 2 except that this increased thickness now becomes the width in Configuration 2 and so Equation 4.3.1 becomes :

$$R_{LIM} / R_O = 1+ 3.2 (w/t)^{0.5} / [1+2.3 g/w + 15(g/w)^2] \qquad 5.1.$$

In this configuration the proximity loss now *increases* with the width w and this is because more of the surrounding magnetic field is intercepted by the adjacent conductor.
Given the above logic it was assumed that a single experiment would be sufficient to validate the above equation, and this equation is potted below along with the measured values :

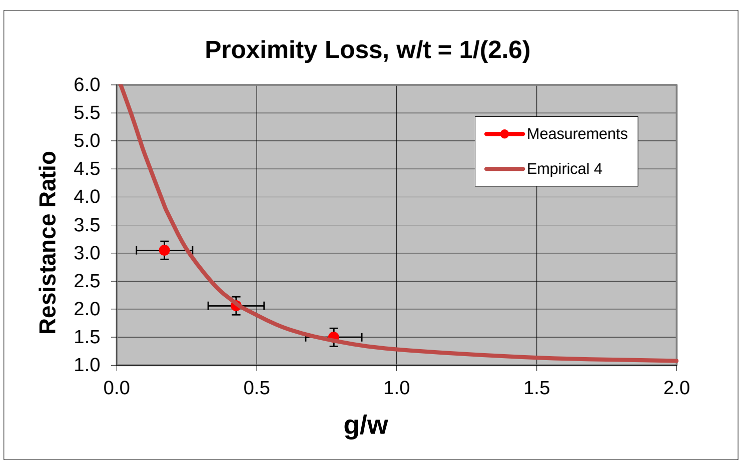


Figure 5.1 Limiting proximity loss Configuration 2, w/t =2.6

A bronze conductor with w/t = 2.6 was used (as in A2.2). It was more difficult to space the conductors accurately in this configuration and so the uncertainty of g/w is therefore larger. Nevertheless the correlation is sufficient to validate the equation.
It should be noticed how very high the proximity loss is in this configuration, with the resistance up to 6 times its value with no proximity loss. The ratio would be even higher for thin planar conductors as found on printed circuits.

6. DISCUSSION

6.1. Alternative forms of Equations 4.3.1.

It is useful to consider alternative forms of Equations 4.3.1. A conductor carrying a DC component needs to have a defined cross-sectional area and so it is useful to express these equations in terms of this area, A. Since A=wt, then t=A/w, so $(w/t)^{0.5} = (w^2/A)^{0.5}$.

Taking Equation 4.3.1 this can then be written as:

$$R_{LIM} / R_O = 1+ 3.2\ A^{0.5} / [w\ (1+2.3\ g/w + 15(g/w)^2] \qquad 6.1.1$$

So for a given cross-sectional area the proximity loss is in inverse proportion to the width.

Alternatively w=A/t, and $(w/t)^{0.5} = (A/t^2)^{0.5}$, and Equation 3.4.1 can then be written as :

$$R_{LIM} / R_O = 1+ 3.2\ t / [\ A^{0.5}\ (1+2.3\ g/w + 15(g/w)^2] \qquad 6.1.2$$

This shows the proximity loss increasing directly with the thickness t for a given area. This is significant because it shows that the proximity loss is proportional to the intercepted magnetic field from the other conductor (ref 1 Appendix 1 for an analysis of circular conductors on this basis).

6.2. Alternative forms of Equations 5.1

Similar equations can be derived from Equation 5.1, and these are given in Section 8.

6.3. Approximation at small and large spacings g/w

Taking the second term in Equations 4.3.1 and 5.1, $(1+2.3\ g/w + 15(g/w)^2$, then at small values of g/w this is approximately equal to 1+2.3 g/w and the slope of R/R_o is approximately proportional to -2.3g/w (since $1/(1+x) \approx (1-x)$ for small x). So the proximity loss reduces directly as the gap g at small gaps.
At large values of g/w the term $(1+2.3\ g/w + 15(g/w)^2$ approximates to $15(g/w)^2$ and so Equation 4.3.1 becomes :

$$R/ R_O \approx 1+ 0.21\ (w/g)^2 / (w/t)^{0.5} \quad \text{for } g/w > 1 \qquad 6.3.1$$

6.4. Comparison with Circular Conductors

It is interesting to compare the above equation with Butterworth's equation for the proximity loss at high frequencies of circular conductors with opposing currents : $R / R_o = 1 + 0.5\ [(d / p)^2 / (1 - 0.75\ (d/ p)^2]$ (ref 1) where d is the diameter of each conductor and p is the pitch between them. When p is very large this approximates to $[1+ 0.5(d/p)^2]$ and since $p \approx g$ at large p then:

$$R/ R_O \approx [1+ 0.5(d/g)^2] \quad \text{for } (d/g)^2 << 1 \qquad 6.4.1$$

Comparing Equations 6.3.1 and 6.4.1, then at large spacings the rectangular proximity loss is proportional to $(w/g)^2$ and the circular loss is proportional to $(d/g)^2$. This favourable comparison gives extra confidence in Equation 4.3.1.
We can anticipate that at large values of g the magnetic field around the rectangular conductor is circular, so it is has the same magnetic intensity as that from a circular conductor of diameter d'. This can be found by equating Equations 6.3.1 and 6.4.1:

$$0.21\,(w/g)^2 / (w/t)^{0.5} = 0.5(d'/g)^2$$

So

$$d' = [0.42/(w/t)^{0.5}]^{0.5}\, w \qquad 6.4.2$$

So for instance at w/t =1 the equivalent diameter for each of the rectangular conductors is d' = 0.65 w (for g/w > 1) to give the same proximity loss.

7. PROXIMITY LOSS VERSUS FREQUENCY

The previous sections have considered the limiting value of the resistance with proximity loss, R_{LIM}/R_O. This resistance is only achieved at very high frequencies, generally much greater than the normal operating frequency for a given cross-sectional area of conductor (ref 2) . In this section the change of R/R_o with frequency is considered, and this has already been determined in Section 3 where the following equation was found :

$$R/R_O = 1 + k\,F_{(f)} \qquad 7.1$$

The limiting value of this equation R_{LIM}/R_O is reached when the frequency is high enough for $F_{(f)}$ to be unity and so $R_{LIM}/R_O = 1+k$ and so $k = R_{LIM}/R_O - 1$, and Equation 7.1 becomes :

$$R/R_O = 1 + (R_{LIM}/R_O - 1)\,F_{(f)} \qquad 7.2$$

8. OVERALL EQUATION FOR RESISTANCE WITH PROXIMITY LOSS

The overall equation for the resistance of two rectangular conductors with opposing currents is the product of Equations 2.1 and 7.2 for Configuration 1, and Equations 5.1 and 7.2 for Configuration 2. Combining these the overall equation becomes :

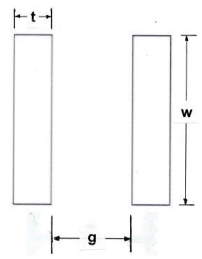
$$\mathbf{R_{ac}/R_{dc} \approx K_C\,(1 + F_{(f)}\,(R_{LIM}/R_O - 1) / (1 - e^{-x})} \qquad \mathbf{8.1}$$

where

$K_C = 1 + F_{(f)}\,[1.2/e^{2.1\,t/w} + 1.2/e^{2.1\,w/t}]$
$F_{(f)} = (1 - e^{-0.026\,p})$
$p = A^{0.5}/(1.26\,\delta)$
A is the cross-sectional area of the conductor
δ is the skin depth = $[\rho/(\pi f \mu)]^{0.5}$
$x = [2\,\delta/t\,(1+t/w) + 8\,(\delta/t)^3/(w/t)] / [(w/t)^{0.33}\,e^{-3.5\,t/\delta} + 1]$
w is the width of the conductor in metres
t is its thickness in metres
f is the frequency in Hz
R_{LIM}/R_O : see below for Configurations 1 and 2
g is the gap between the conductors, measured face to face

NB In K_C the factor $1.2/e^{2.1\,w/t}$ is extremely small for w/t above 3, and indeed at w/t above about 300 Excel is unable to calculate the value and returns an error message.

R_{LIM} / R_O for Configuration 1 :

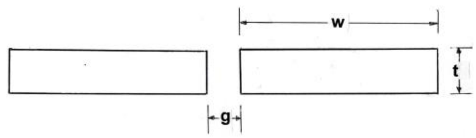


$$R_{LIM} / R_O = 1 + 3.2 / [(w/t)^{0.5} (1 + 2.3\, g/w + 15(g/w)^2\,] \qquad \textbf{8.1a}$$

$$= 1 + 3.2\, A^{0.5} / [w (1 + 2.3\, g/w + 15(g/w)^2] \qquad \textbf{8.1b}$$

$$= 1 + 3.2\, t / [\, A^{0.5} (1 + 2.3\, g/w + 15(g/w)^2] \qquad \textbf{8.1c}$$

R_{LIM} / R_O for Configuration 2 :



$$R_{LIM} / R_O = 1 + 3.2\, (w/t)^{0.5} / [\, 1 + 2.3\, g/w + 15(g/w)^2\,] \qquad \textbf{8.1d}$$

$$= 1 + 3.2\, w / [A^{0.5} (1 + 2.3\, g/w + 15(g/w)^2] \qquad \textbf{8.1e}$$

$$= 1 + 3.2\, A^{0.5} / [\, t (1 + 2.3\, g/w + 15(g/w)^2] \qquad \textbf{8.1f}$$

The above equation is shown below for w/t =1 and g/w = 0.05 along with the measurements. The equation is shown in green and the measured values in blue :

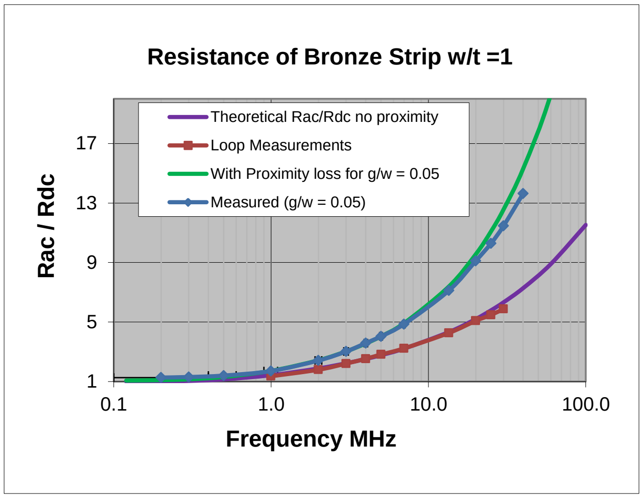


Figure 8.1 Comparison of equations with measurements

The difference between these two curves is due partly to experimental error and partly to errors in Equation 8.1. However the equation is likely to be the more accurate since it has been chosen to agree with all the measurements taken, and not just this one.
Also shown are the curves for the resistance ratio *without* proximity loss, with Equation 2.1 plotted in purple and the measurements in brown.

Appendix 1 : MEASUREMENT APPARATUS

A1.1. Conductors

Very short conductors had to be used in order to raise the SRF (see Appendix 3), and this meant that the resistance to be measured was very low. Conductors of bronze and nichrome were therefore used because they have a higher resistivity than copper.
The conductors to be measured were cut in the middle and the two strips were placed back to back as shown in the photo below :

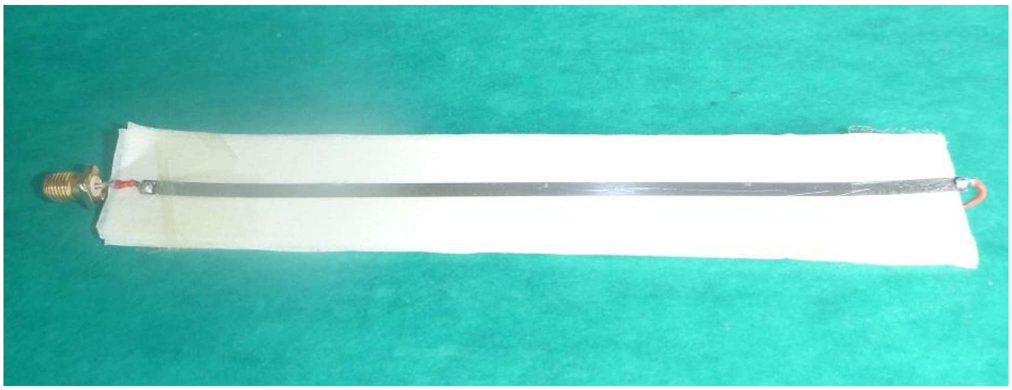


Figure A 1.1.1 Nichrome strips with spacer

This photograph shows one nichrome strip mounted on a spacer which separated it from the strip underneath. The spacer was made from layers of masking tape to the required thickness. At the right- hand end is a short red wire connecting together the two strips and at the other end an SMA connector connected to both strips by short red leads. The resistance of these leads was subtracted from the measurements. All joints were soldered.
The bronze conductors were more flexible and so could be folded in the middle.
The nichrome strips (w/t = 10.9) had fairly sharp corners. The square bronze conductor (w/t=1) had rounded corners with a radius estimated as 1/6 th the thickness. The bronze conductor with w/t = 2.6 had a very rounded edge so that its cross-section was a somewhat 'stadium' shaped, and the width was measured as the maximum value across the stadium.

A1.2. Loss in spacer and clamps

Pressure was applied by plastic pegs (not shown) to keep the strips close to the spacer. There will be additional loss due the spacer and the clamps, and to assess this two *copper* conductors were placed back to back and the loss measured firstly with an air gap between them and then with the 0.97 mm thick spacer and plastic clamps. The resistance at 75 MHz increased from 0.216 to 0.222 Ω, so these losses were negligible.

A1.3. Measurement Apparatus

All resistance measurements were made with an Array Solutions UHF Vector Network Analyser. Calibration of this analyser required an open circuit, a short circuit and known resistive load, and these are shown below.

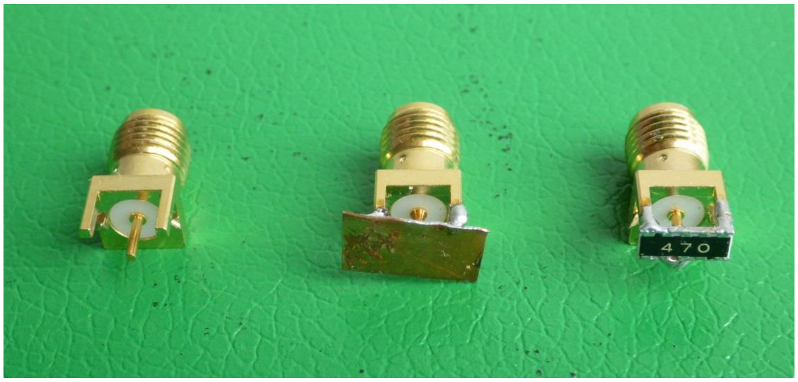


Figure A 1.3.1 Calibration loads

To ensure that the calibration resistance had minimal stray reactance a thick-film resistor was used (above), and this had the added advantage that it could be located in the same plane as the short circuit. Its value was 47 Ω ± 1%. SMA connectors were used because they are small and therefore have a small stray capacitance, and so any error in calibrating this out would also be small.
The resistance values measured here were very low : generally less than 2 Ω, and often less than 1Ω. To give a measure of the accuracy at these values a ¼ w metal film resistor with a measured dc resistance of 2.21 Ω was measured for frequencies up to 100 MHz. The measured values were greater by between 0.1 Ω and 0.2 Ω and this error was subtracted from all measurements.

Appendix 2 MEASUREMENTS

A2.1. w/t =1

A bronze conductor was used here with a measured resistivity of 1.6 10^{-7} Ω m. The width was equal to the thickness and measured as 0.66 mm. The total length was 0.616 m and the conductor was folded back on itself for the measurements.
The first measurement was that of the loop (ie folded but with wide spacing, Section 4) but this did not compare well with the theory, Equation 2.2. Previous measurements with bronze wire had shown that impurities gave a permeability greater than unity, and that these impurities were near the surface and so the effective permeability changed with skin depth. It was found that if the relative permeability was increased exponentially with frequency to a value of 1.45, according to the following equation then the measurements agreed with the theory :

$$\mu_R = 1 + 0.45\,(1 - e^{-f}) \qquad \text{A 2.1}$$

where f is the frequency in MHz

The measured values, extrapolated to the limiting value, are shown in Figure 4.3.1.

A2.2. w/t = 2.6

A bronze conductor was used here with width 1.27 mm, thickness 0.49 mm and length 0.610 mm, so the folded length was 0.305 mm. Resistivity was measured as 1.6 10^{-7} Ω m. Unlike that for w/t =1 above the ‘loop’ measurements agreed well with the theory and the permeability was therefore unity over the whole frequency range.
The measured values, extrapolated to the limiting value, are shown in Figure 4.3.2.

A2.3. **w/t =10.9**

A nichrome strip was used here with a width of 3.05 mm and a thickness of 0.28 mm. Two lengths were cut and the ends tinned with solder, giving an active length of nichrome between the edges of the solder of 0.146 m. The DC resistance of the two in series was 0.262 Ω giving it a resistivity of $0.777\ 10^{-6}\ \Omega$ m (this is less than the normally quoted range of 1 to $1.5\ 10^{-6}\ \Omega$ m). The relative permeability of pure nichrome is unity but measurements indicated a raised permeability, probably because of impurities especially iron (see Payne ref 1), confirmed by the fact that the strip was attracted by a permanent magnet. This raised permeability introduced an added complication but had the advantage that it reduced the skin depth so measurements could be made at lower frequencies.
However the permeability was frequency dependent and good agreement with the 'loop' measurements was achieved with the permeability according to the following :

$$\mu_R = 1 + 2\,(1 - e^{-0.2\,f}) \qquad \text{A 2.3.1}$$

where f is the frequency in MHz

So the maximum permeability was 3.
The measured values, extrapolated to the limiting value, are shown in Figure 4.3.3.

A2.4. **w/t = 15.3**

A bronze conductor was used here with width 2.45 mm, thickness 0.16 mm and length 0.598 mm, so the folded length was 0.305 mm. Resistivity was measured as $1.09\ 10^{-7}\ \Omega$ m. The 'loop' measurements agreed well with the theory and so the permeability was therefore unity over the whole frequency range.
The measured values, extrapolated to the limiting value, are shown in Figure 4.3.4.

A2.5. **Error analysis**

The results shown in Section 4.3 include error bars and these have been assessed as follows.
The uncertainty in g/w (x axis) is due to the uncertainty in the values of g and w. Taking g, this was equal to the thickness of the spacer and the degree to which the strips were held against it. The micro-meter which measured the spacer had an accuracy of ±0.02 mm and in addition there was an uncertainty of the spacing dependent on the mechanical pressure applied, which was estimated as -0+0.1 mm. The resultant error in g assumes these errors are uncorrelated to give an uncertainty of -0.02 to +0.1mm. Here we are interested in g/w so the uncertainties in g are reduced by w as follows :

w/t	w (mm)	g/w uncertainty
1	0.66	0 to +0.15
2.6	1.27	0 to + 0.08
10.9	3.05	0 to + 0.03
15.3	2.54	0 to + 0. 04

NB there is also an uncertainty in w but this is small compared with that of g and can be ignored.

In assessing the errors in the resistance measurements (the y axis) it is important to note that the requirement here was to measure the *change* in the resistance due to proximity. This has been measured as the increase in resistance from that of the conductor with essentially no proximity (Section 2) so absolute accuracy was not important.
To determine the errors in the *relative* accuracy an analysis of the data for Figure A1.2.3 shows the measurement points deviating by ± 3% and to this must be added the uncertainty in the length of the conductor of ± 3% giving an overall uncertainty of ± 4.2% in resistance. This translates to an uncertainty in resistance ratio of ± 0.08 for the average resistance ratio of 2. To this must be added the uncertainty in extrapolating the measurements to the limiting value of the proximity loss. This is assumed to be equal to the uncertainty of the fit to the exponential curve over the measurement range multiplied by the ratio of the extrapolation to the measurements. Generally the fit over the measurement range of R/R_o is ± 0.05, and the

extrapolation is generally by a factor of 2.8 and so the uncertainty in the extrapolation is ± 0.14 giving an overall uncertainty in R_{LIM} / R_O of ± 0.16.
There will be errors in the ratio w/t and these will affect the accuracy of Equations 4.3.1 and 5.1. The measurement errors in w and t will cancel in the ratio w/t, and the main error in this ratio is the degree to which each conductor is rectangular. This was essentially true for all the conductors used except the bronze with w/t = 2.6, which had a semi stadium-shaped cross-section. The width across the whole stadium was 1.29 mm, and it was assumed that this was equivalent to a rectangular conductor of width 1.24mm
The uncertainty in the frequency measurement is negligible.
Because the measurements were calibrated against a loop conductor the effects of temperature on the resistivity were cancelled.
The nichrome strip was much wider than the wires feeding it and so there will be an uncertainty in the effective length due to the current fanning-out into the strip. However this effect is considerable reduced because the ends of the strips were soldered across their width and the current will have fanned-out in the solder, given that solder has a resistivity of around ¼ that of the nichrome.

Appendix 3 : SELF-RESONANCE

The wire when folded constituted a two-wire transmission-line and this resonates when its length is equal to nλ/4, where λ is the wavelength. Thus the first resonant frequency is when

$$f_1 = 300/ (2\,\ell) \qquad \text{A 3.1}$$

where ℓ is the total length of the wire (ie twice the length of the line).
f_1 is in MHz

As this frequency is approached the measured resistance and the inductance increase above their low frequency value and Welsby (ref 4, p 37) has shown that the low frequency value is given by :

$$L = L_M [1- (f / f_R)^2] \qquad \text{A3.2}$$

$$R = R_M [1- (f / f_R)^2]^2 \qquad \text{A3.3}$$

L_M and R_M are the measured values, and f_R is the self-resonant frequency. Welsby developed these equations for a lumped element tuned circuit and to test the accuracy of Equation A3.3 with the transmission-line formed by the conductors, the resistance of two parallel conductors was measured with a well-defined known resistance. This was achieved by terminating the two conductors with a thin film resistor of measured DC resistance of 8.68 Ω. The conductors consisted of two parallel copper strips of width 6mm and thickness 0.32 mm and length of 150 mm with a spacing of about 8 mm. The measured SRF was 239 MHz and the change in resistance with frequency calculated from the above equation, and plotted below in blue.

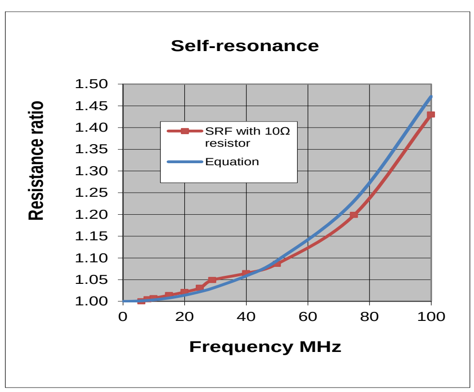


Figure A 3.1 SRF comparison

The resistance was then measured and this is plotted in red. The discrepancy compared with Equation A3.3 was less than 3%. This was reduced to less than 1% if the SRF was assumed as 250 MHz

However despite this good comparison a problem often encountered, especially with the larger spacings, was that there was a spurious partial resonance at a lower frequency than the main resonance. The reason for this is not known, but when present this further limited the maximum measurement frequency which could be used and it was found in practice that corrections due to SRF of greater than 10% did not give reliable results.

Self-resonance severely constrained the measurements here and the author intends to make a special study of this in the future.

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