SKIN EFFECT, PROXIMITY EFFECT AND THE RESISTANCE OF CIRCULAR AND RECTANGULAR CONDUCTORS

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SKIN EFFECT, PROXIMITY EFFECT AND THE RESISTANCE OF CIRCULAR AND RECTANGULAR CONDUCTORS

Simple but accurate equations are given for the AC resistance of a single circular conductor and for parallel pairs of conductors with current flowing in the same direction and in opposite directions. These equations are extended to multiple wires side by side and carrying current in the same direction, and this gives the basis for determining the resistance of rectangular conductors.

1. INTRODUCTION

The AC resistance of a single circular conductor is surprisingly difficult to analyse. The solution involves Bessel functions but these are difficult to handle and so the problem is often simplified by assuming that the current flows in a thin skin around the periphery, but this fails at low frequencies. The author has determined a simple equation which applies at all frequencies and is surprisingly accurate.

When two such wires are brought close to one another the resistance of both wires increases, and this increase is dependent upon the relative directions of the currents flowing in the two wires. When the wires carry currents in the same direction the resistance can increase by up to 35%, depending upon the spacing of the wires. When the currents are in opposite directions the increase is very much higher, and some theories give the resistance approaching infinity at very close spacing.

In this paper the theories for twin wires are considered and their accuracy assessed against experiments. These theories are then extended to cover the resistance of multiple wires placed side by side, to form a sheet of wires. The resistance profile across this sheet is a good approximation to that of a flat strip conductor, permitting its resistance to be calculated. This is detailed in reference 13.

In presenting the resistance of the various configurations it is convenient to express this as the ratio to that of the straight wire in isolation R/R_o, since this ratio gives the increase due to proximity effects.

2. SINGLE CIRCULAR CONDUCTOR

2.1. Introduction

Current flowing in a wire produces a magnetic field, and lines of constant magnetic intensity are shown below .

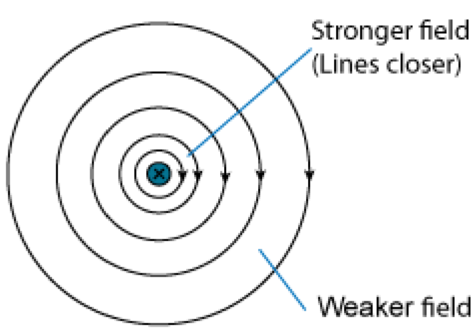

Figure 2.1.1 Lines of constant Magnetic Intensity H around a wire carrying a Current

At high frequencies the magnetic field within the conductor (not shown above) causes the current to flow in the outer periphery in a thin skin, and so this effect is known as the ‘skin-effect’.

Skin-effect in cylindrical conductors is discussed below, starting with the simpler problem of skin-effect in a flat conductor.

2.2. Skin Effect in Wide Flat Conductor

In a wide flat conductor, the current which is set-up on the surface diffuses into the surface exponentially according to the resistivity of the material, its permeability and the frequency. For a current density J_o at the surface, the density at depth z is given by (see Ramo & Winnery ref 2, p237) :

$$J_z = J_o\, e^{-z/\delta} \qquad 2.2.1$$

where $\delta = [\rho/(\pi f \mu)]^{0.5}$

$\mu = \mu_r\, \mu_o$

μ_r is the material relative permeability (= 1 for copper)

$\mu_o = 4\pi\, 10^{-7}$

ρ = resistivity (ohm-metres) (= $1.68\, 10^{-8}$ for copper)

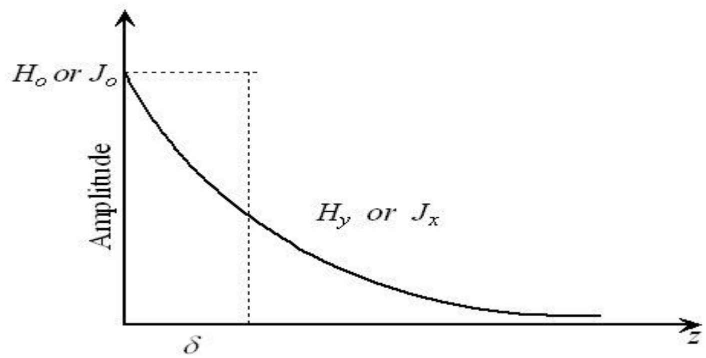

Figure 2.2.1 Current density at high frequencies

Thus the current density decays exponentially as shown in the above curve, and so the *total* current is equal to the area under this curve, integrated from the surface to the material thickness t ie from z = 0 to z = t :

$$I_{av} = J_m \int_0^t e^{-z/\delta}\, dz \qquad 2.2.2$$

$$I_{av} = J_m \left[-\delta\, e^{-z/\delta}\right]_0^t = J_m \left[-\delta\, e^{-t/\delta} - (-\delta)\right]$$

$$\mathbf{I_{av} = J_m\, \delta\, (1 - e^{-t/\delta})} \qquad \mathbf{2.2.3}$$

When the conductor thickness is infinite, the exponential function is zero and the average current becomes $I_{av} = J_m\, \delta_1$, This has the same area as that of a current uniformly distributed down to depth of δ, and zero at greater depths and this leads to the definition of skin depth :

$$\textbf{Skin depth } \delta_\infty = [\rho/(\pi f \mu)]^{0.5} \qquad \mathbf{2.2.4}$$

Although the skin depth defined above has been calculated assuming an infinite thickness of conductor, Equation 2.2.3 shows that the current density at two skin depths has dropped to e^{-2} = 0.14 or to a power of about 2% of that at the surface. So this suggests that Equation 2.2.4 applies as long as the thickness of the conductor is greater than two skin depths. However Wheeler (ref 1) says that this is true for a metal screen where penetration is from one side only, but in a flat conductor penetration is from two sides and then the thickness must be four skin depths for Equation 2.2.4 to be valid. However an experiment by the author

(Section 6.4.1) does not support Wheeler's statement (this experiment is in support of Equation 2.2.5 below, but the same arguments would apply). See also Section 2.5.

The skin depth δ_∞ in mm for various metals is given below (Ramo & Whinnery ref 2 p240). The frequency f is in Hz.

Copper	:	$66.6 / \sqrt{f}$
Aluminium	:	$83 / \sqrt{f}$
Brass	:	$127 / \sqrt{f}$
Nichrome resistance wire	:	$500/ \sqrt{f}$
Mu metal	:	$4 / \sqrt{f}$

The skin depth in copper for frequencies from 1 KHz to 1000 MHz is shown below.

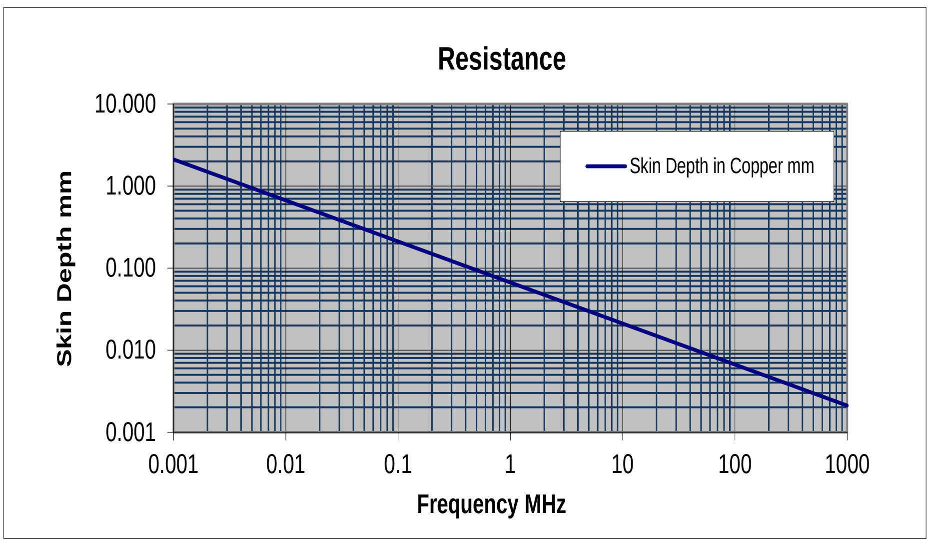

Figure 2.2.2 Skin Depth in Copper

2.3. Cylindrical Shell Approximation

The skin-effect in a circular conductor has been analysed by Ramo & Whinnery (ref 2 p243), and they show that the distribution of current involves complex Bessel functions Ber and Bei (i.e. real and imaginary):

$$i_z = i_o \,(\mathrm{Ber}\,\sqrt{2}\, r/\delta + j\,\mathrm{Ber}\,\sqrt{2}\, r/\delta) \,/\, (\mathrm{Ber}\,\sqrt{2}\, r_o/\delta + j\,\mathrm{Ber}\,\sqrt{2}\, r_o/\delta \qquad 2.3.1$$

where r_o is the radius of the conductor and r is the radius of the field

The solution to this equation is complicated but if the skin-depth is small compared to the conductor diameter so that the effect of the curvature is small, then an exponential decay as Equation 2.2.1 is a good approximation. In that case it can be assumed that the current flows in a hollow cylindrical shell having the same outside diameter as the wire and having a thickness δ :

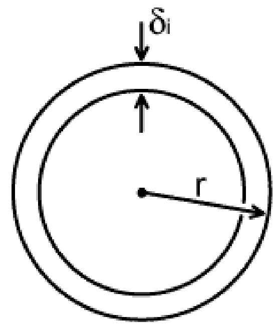

Figure 2.3.1 Skin depth at high frequencies

The resistance of such a tube will be equal to $R = \rho\, \ell/A$, where A is the area of the conducting cross-section. So for a wire of outside radius r_w :

$$R_o \approx \rho\, \ell /[\{\pi\, r^2_w\} - \{\pi\, (r_w - \delta)^2\}]$$

$$= \rho\, \ell /[\pi\, (d_w \delta - \delta^2)] \qquad 2.3.2$$

It is useful to express this HF resistance in terms of the direct current resistance $R_{dc} = \rho\, \ell/A = 4\, \rho\, \ell/(\pi\, d^2_w)$. which then becomes :

$$R_{ac} / R_{dc} \approx 0.25\ d^2_w /(d_w \delta - \delta^2)] \qquad 2.3.3$$

Dividing top and bottom by δ^2 puts this equation in terms of d_w/δ, which is sometimes more useful:

$$R_{ac} / R_{dc} \approx 0.25\ (d_w / \delta)^2\ /(d_w/\delta - 1)] \qquad 2.3.4$$

where $R_{dc} = 4\, \rho\, \ell/ \{\pi\, (d_w)^2\}$
ρ is the resistivity ($1.72\ 10^{-8}$ for copper at 20^0 C)
ℓ is the length in meters

Notice that when d_w/δ tends towards unity the above equations tend towards infinity and therefore fail. These equations are plotted below in green, along with the accurate values of Equation 2.3.1 as tabulated by Terman (ref 3, p31) in blue.

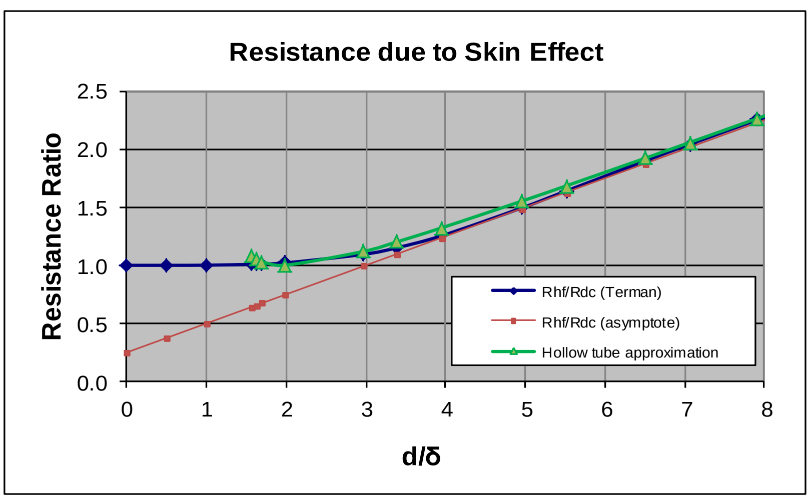

Figure 2.3.2 Resistance of Round Wire at High frequencies

The above equations agree with the tabulated values to within ±5.5% for all values of $d_w/\delta \geq 1.6$.
Also shown is the asymptote of Equation 2.3.1 for large values of d_w/δ, and this is given by :

$$R_{ac} / R_{dc} \approx (d_w/\delta + 1)/4 \qquad 2.3.5$$

The error is less than -1.3% for all values of d_w/δ above 4.

2.4. Exponential Approximation

The cylindrical shell approximation above fails when d_w/δ is less than 1.6 ie at low frequencies. The empirical equation given below is more accurate and does not fail at low frequencies :

$$\mathbf{R_o / R_{dc} = 1/(1 - e^{-x})} \qquad \mathbf{2.4.1}$$

where $\mathbf{x = 3.9/(d'/\delta) + 7.8/(d'/\delta)^2}$
d' is the receded diameter (see below)

This equation is derived in Appendix 6, and has an accuracy of better than ± 3.6% for any value of d'/δ.
It is plotted below along with the accurate tabulation by Terman :

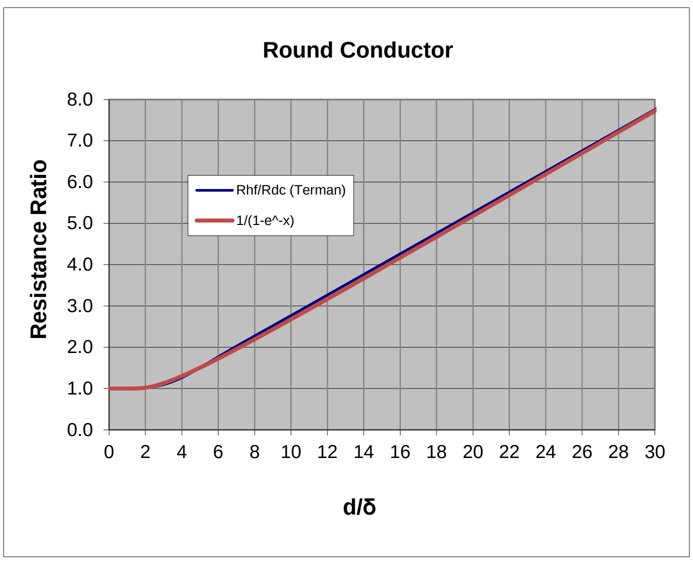

Figure 2.4.1 Round Wires : Empirical equation

2.5. Current Recession

The diameter of a circular conductor is normally taken as its physical diameter. However for the calculation of inductance, capacitance or resistance at high frequencies it is the diameter at which the current flows which is needed, and the current recedes from the surface by half a skin depth (see Wheeler ref 1) so that the diameter of current flow is :

$$\mathbf{d' = d_w - \delta} \qquad \mathbf{2.5.1}$$

where $\mathbf{d_w}$ **is the physical diameter**
d is the receded diameter

Where current recession applies the equations are given in terms of d' rather than d_w.

2.6. EM Wave or Diffusion ?

Associated with the exponential reduction in amplitude (Equation 2.2.1) is a change of phase, with an angle of one radian at one skin depth. The penetration into the conductor thus has a wave-like characteristic and indeed it is normally described as the penetration of an *EM wave* into the conductor. However there is an alternative view, and Spreen (ref 10) shows that Equation 2.2.1 can also describe diffusion, which is defined as the net movement of a substance from a region of high concentration to a region of low concentration. Given that conduction in metals is a movement of charged particles (electrons), diffusion seems to the author to be a more likely mechanism.
To resolve this issue the author has conducted experiments and shown that the penetration is due to diffusion (ref 12).

3. PROXIMITY LOSS IN TWIN WIRES – CURRENTS IN SAME DIRECTION

3.1. Introduction

When two parallel wires carry current in the same direction the magnetic field intensity H is shown below (compare with that of the single wire Figure 2.1.1) :

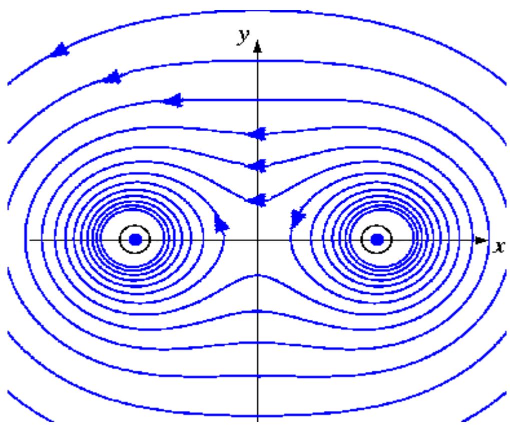

Figure 3.1.1 Lines of constant H around Two parallel Wires carrying Similar Currents

3.2. Theory

It should be noted that in the above figure the field is identical to that of the equi-potential lines around two line charges, and also that the central lines are approximately circular. These attributes are used in Appendix 1 to derive the following simple equation for high frequencies:

$$R / R_o \approx 1 + [(1/r'_1) - (1/r'_2)] \qquad 3.2.1$$

where $r'_1 = (2p/d' + 0.5\, d'/p - 1)$
$r'_2 = (2p/d' + 0.5\, d'/p + 1)$
R_o **is given by Equation 2.4.1.**
$d' = d_w - \delta$

Other than the author's analysis above the only other known is that by Butterworth. He gave a very complicated analysis in 1921 (ref 4), which would apply to all frequencies (not just to high frequencies) resulting in the following equation (his equation 48) :

$$R = R_{dc}\,[1+F + G\,(d_w / p)^2 / (1- (d_w / p)^2\, H)] \qquad 3.2.2$$

R_{dc} is the dc resistance
F, G and H are tabulated by Butterworth

The factor R_{dc} [1+F] is the resistance of the isolated wire, due to skin effect alone. The factor G is due to the induced eddy currents, and at high frequencies G = 0.5 (1+F). Also at high frequencies he gives
H = -0.25. So at high frequencies the above equation becomes, in terms of the ratio to the resistance of the isolated wire R_o, and including recession :

$$\mathbf{R / R_o = 1 + 0.5\,[(d' / p)^2 / (1+ 0.25\,(d' / p)^2]} \qquad \mathbf{3.2.3}$$

where $d' = d_w - \delta$

This equation is plotted below (green) along with the author's equation, Equation 3.2.1 (blue) :

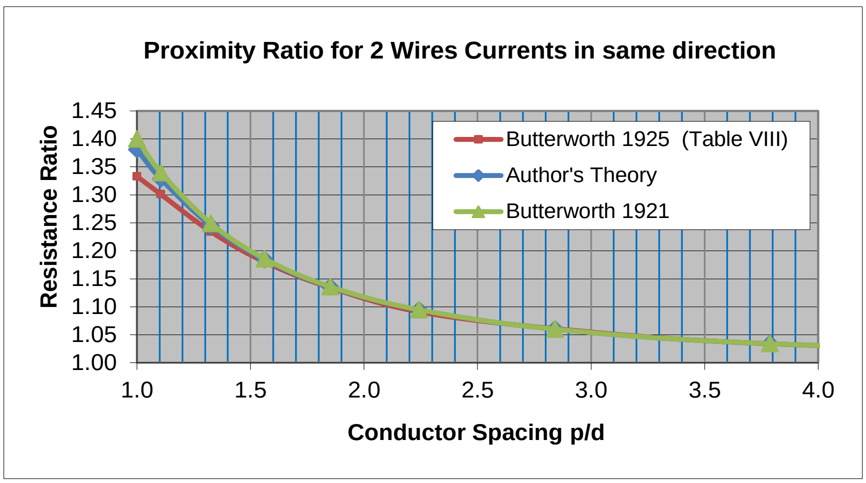

Figure 3.2.1 Theoretical Resistance Ratio for Currents in Same Direction

The agreement is within 1.4%. In 1925 Butterworth published another complicated analysis (ref 5) and his final equation is difficult to use because it contains elliptic integrals. However he tabulated the results (his Table VIII) for high frequencies and these are plotted above in red. An empirical equation which fits this data within ±1% is given below and has the advantage of simplicity:

$$\mathbf{R / R_o = 1 + 1/ (2 x^2 +1)} \qquad \mathbf{3.2.4}$$

where $x = (p/ d')$
p is the distance between wire centres
$d' = d_w - \delta$

NB Notice that Butterworth's earlier equation (Equation 3.2.3) can be written as $R / R_o = 1 + 1/(2x^2+ 0.5)$.

The above equations are for high frequencies only, where the skin depth is much less than the conductor diameter.

3.3. Experimental Support

3.3.1. General

The following experiments give support to the above equations, however it has not been possible to determine the most accurate because the difference between them is very small (all are within ± 2%).
The ac resistance of the following configurations were measured :

a) Two wires insulated, twisted together, and connected at each end.
b) As above but un-insulated and connected at each end.
c) As a) but one wire disconnected.

The wires were twisted as a method of keeping them close together down their whole length, and this twisting was shown not to have affected the resistance measurements.

3.3.2. Resistance Ratio : Currents in Same Direction

Appendix 2 describes an experiment to measure the increased loss in a pair of copper wires, 6 m long, and carrying current in the same direction. In the figure below the measured results are compared with the values calculated from Equation 3.2.4 :

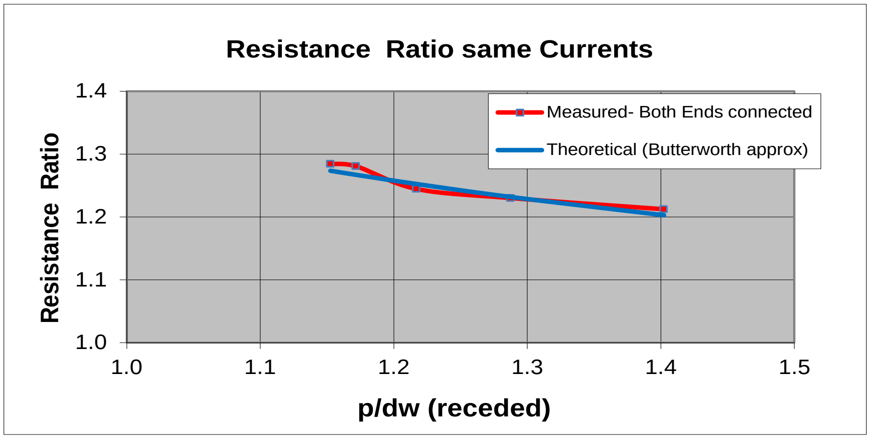

Figure 3.3.2.1 Resistance Ratio for Currents in Same Direction

The correlation is seen to be very good and within ±1%. The above graph shows a range of wire spacings p/d_w, but the wires in the tests had only one *physical* value of p/d_w, and this was 1.08 set by the thickness of the enamel insulation. However at the frequencies used here, the effective wire diameter reduces by the skin depth (see Section 2.4), and this increases p/d_w to value depending upon the measurement frequency.

3.3.3. The Effect of Twisting

In the experiment above the two wires needed to be kept close together down their whole length and this was achieved by twisting them together. It has been reported that twisting can increase the resistance, and so to test this additional twisting was tried but it had no effect on the results (see Appendix 2).

3.3.4. Un-insulated Conductors

The above experiment used insulated wire, and it raises the question as to whether the same result would be achieved with un-insulated wire. Unfortunately bare copper magnet wire is not readily available, and it is difficult to remove the insulation without damaging the wire. There are chemical methods available using heated sodium hydroxide but this is not practical for the very long length which would be needed to make the resistance high enough to be measurable (eg 6 metres). The alternative was to use Nichrome wire, which is available un-insulated, and has the added advantage that it has a much higher resistivity than copper wire (around 70 times), so that a much shorter length can be used. However it transpires that impurities in its manufacture mean that its skin depth is difficult to calculate, making it impossible to compare measurements with theory. However this does not preclude its use in comparing different wire configurations, since the skin depth will be same in each. These skin depth problems are discussed in Appendix 4.

For the measurements, two Nichrome wires of diameter 0.72 mm and length 0.97 m were used, and one was measured as the reference, and then twisted with the other. The two wires were connected together at each end with a screw terminal from a mains-wire connector. For comparison two more wires were twisted together but one had been insulated with a thin layer of varnish.

The measured results are shown below with the insulated pair in red and the un-insulated pair in dark blue. The resistance of these pairs has been doubled so that comparison can be made with that of the single wire, shown in light blue.

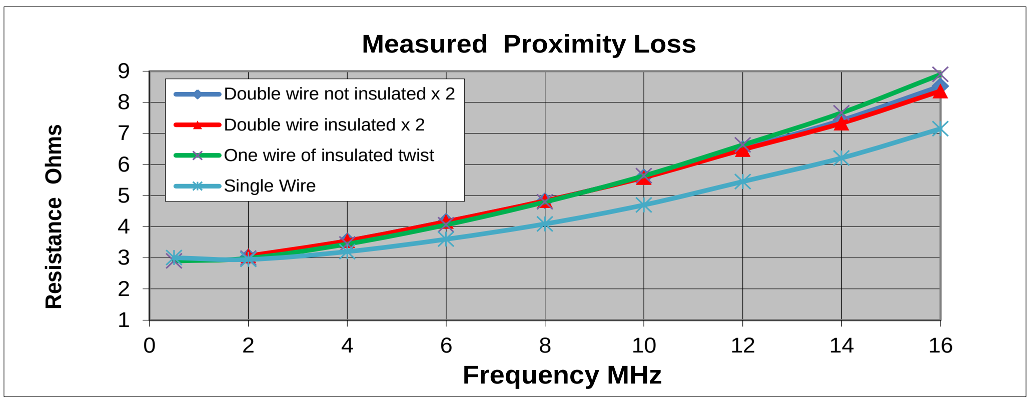

Figure 3.3.4.1 Measured Resistance: Currents in Same Direction

Clearly the insulation makes no significant difference.

Also shown in green is the resistance of one wire of the insulated pair, with the other *open circuit* ie not connected to the other wire at their ends. The ac resistance is essentially the same as when both wires were connected together (red curve).

The rapid rise in resistance with frequency is due mainly to the effects of self-resonance (see Appendix 7) and this was assessed to be around 30 MHz, although with the high loss of theses wires a clear resonance was not present. The SRF was therefore very close to the maximum measurement frequency, and a small difference in the SRF between the configurations probably accounts for the difference between the curves at high frequencies.

3.3.5. Variation of Proximity Loss with Frequency

Equation 3.2.4 gives the resistance of two parallel wires at high frequencies. The change in resistance with frequency has already been measured, and is shown in Figure 3.3.4.1. This shows the resistance of a single isolated wire, and its increase in resistance when another wire is placed along-side it, with only a small gap between them due to the insulation (notice that they do not have to be connected). The ratio of these resistances is plotted below in blue, with the horizontal axis equal to ratio of the skin depth to the wire diameter δ/d_w :

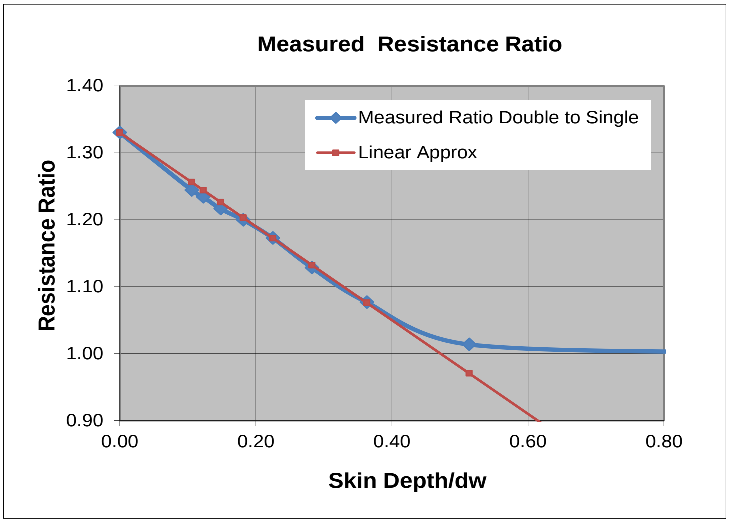

Figure 3.3.5.1 Measured proximity effect of two wires: currents in same direction

It is seen that the extra loss due to proximity of two wires has a maximum value of 1.33 when δ/d_w is very small as predicted by Equation 3.2.4. Also shown (in red) is the following empirical equation :

$$R/R_{dc} = 1.33 - 0.7\ \delta/d_w \qquad 3.3.5.1$$

It is seen that the proximity loss starts to rise when the value of δ/d_w is about 0.5. This is similar to the transition for skin effect in a round conductor which occurs when δ/d_w is about 0.33.

An empirical equation which better matches the measured the whole curve is :

$$R/R_{dc} = 1.33 - 0.9\ (d_w/\delta\ -0.63)/\ (d_w/\ \delta)^2 \qquad 3.3.5.2$$

This is plotted below :

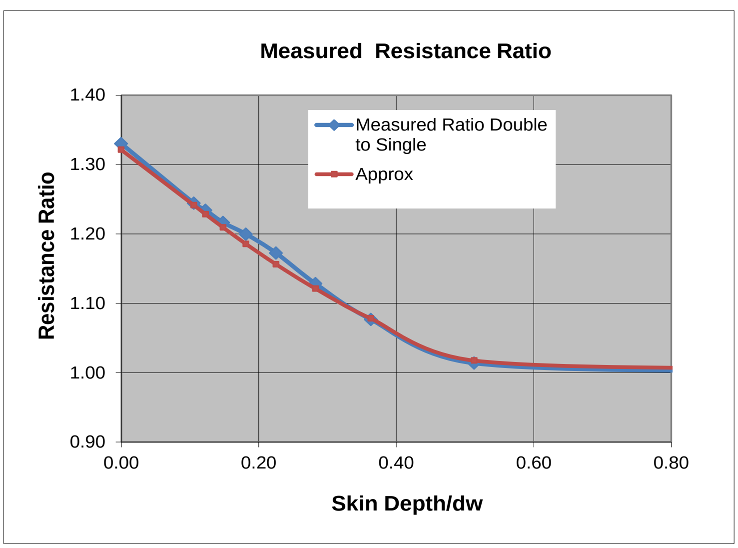

Figure 3.3.5.2 Empirical equation for proximity loss

Note that the above equation fails when $d_w/\delta = 0.63$ (ie $\delta/d_w = 1.6$).

3.3.6. Summary and Conclusions

The experiments with copper and Nichrome wires show that for a straight pair of wires each carrying current in the same direction:

- Each wire in a twin has a higher ac resistance than the single wire, and this increase is consistent with the Equation 3.2.4
- Insulation between the wires makes no difference
- If the second wire is disconnected from the first, the proximity increase is unchanged (the wires being insulated).
- Equation 3.2.4 assumes that the skin depth is much smaller than the conductor diameter but the equation can be used at larger skin depths (ie lower frequencies) if the current is assumed to recede into the wire surface (Section 2.4).
- The proximity loss given by Equation 3.2.4 applies at high frequencies where the skin depth is very small compared to the wire diameter. At lower frequencies the proximity loss is lower (zero at dc of course) with the proximity loss beginning to rise at $d_w/\delta \approx 2$.

4. PROXIMITY LOSS IN TWIN WIRES – OPPOSING CURRENTS

4.1. Theory

When two parallel wires carry current in opposite directions the lines of constant magnetic intensity H are as shown below :

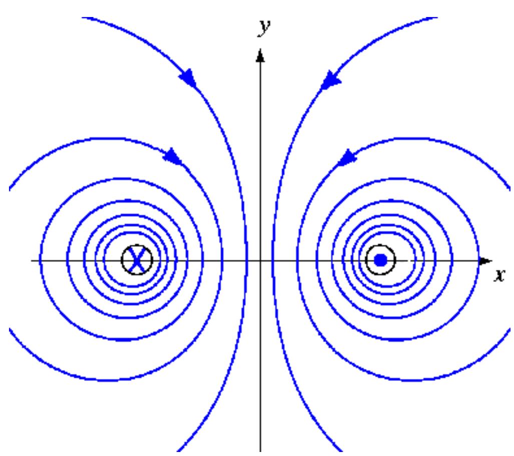

Figure 4.1.1 Lines of constant H around Two parallel Wires carrying Opposing Currents

Appendix 1 shows that the resistance of each wire at high frequencies is then given by :

$$\mathbf{R / R_o \approx 1 + [(1/r'_1) - (1/r'_2)]} \qquad \mathbf{4.1.1}$$

where $r'_1 = (2p/d' - 0.5\, d'/p - 1)$
$r'_2 = (2p/d' - 0.5\, d'/p + 1)$
R_o is the resistance of an isolated single wire
$d' = d_w - \delta$

Butterworth's equation (Equation 3.2.2) is also applicable and he gives H = 0.75 for opposing currents at high frequencies, giving :

$$\mathbf{R / R_o = 1 + 0.5\, [(d'/p)^2 / (1 - 0.75\, (d'/p)^2]} \qquad \mathbf{4.1.2}$$

Moullin (ref 6 p253) gives the following equation for high frequencies, based upon the theory of images :

$$\mathbf{R / R_o = 1/ [1- (d'/p)^2]^{0.5}} \qquad \mathbf{4.1.3}$$

Wheeler (ref 1) also gives the same equation, but his derivation seems to be totally different in that it utilises the equation for the *inductance* of twin wires, and the resistance calculated from his 'incremental inductance rule'. These equations are plotted below :

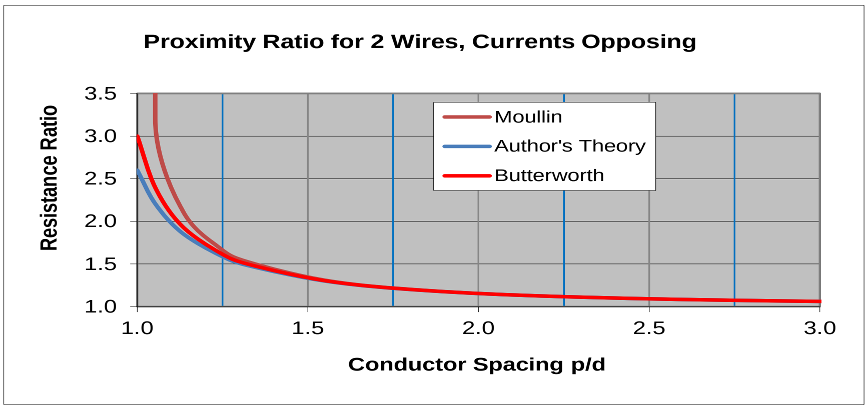

Figure 4.1.1 Theoretical Resistance Ratio for Opposing Currents

Equation 4.1.3 goes to infinity when p/d =1, whereas the other two equations have a fixed value albeit not the same value. A value of infinity seems rather unlikely : if we take for example a very close spacing of p/d =1.03 then Equation 4.1.3 gives R/R_o = 4.2, and if the spacing is reduced by a very small amount so that p/d_w =1 then the equation says that we should expect the resistance to rise to infinity.

4.2. Experimental Support for the Theories

The equations are within 5% of one another for p/d_w greater than 1.25, and so to distinguish between them any experiment must measure the proximity loss for p/d_w ratios less than this. However such close spacing is very difficult to achieve using commercially available magnet wire because of the thickness of the insulation, and given this problem the measurements were inconclusive (see Appendix 3).
To achieve closer spacing the enamel insulation was stripped by heating the wire in a gas flame, and this had the added advantage of producing a very thin oxide coating which provided the necessary insulation. This oxide layer was extremely thin (10-50 microns) and so the effective p/d_w ratio was determined mainly by the recession of the current into the surface. This experiment gave the following results (Appendix 3):

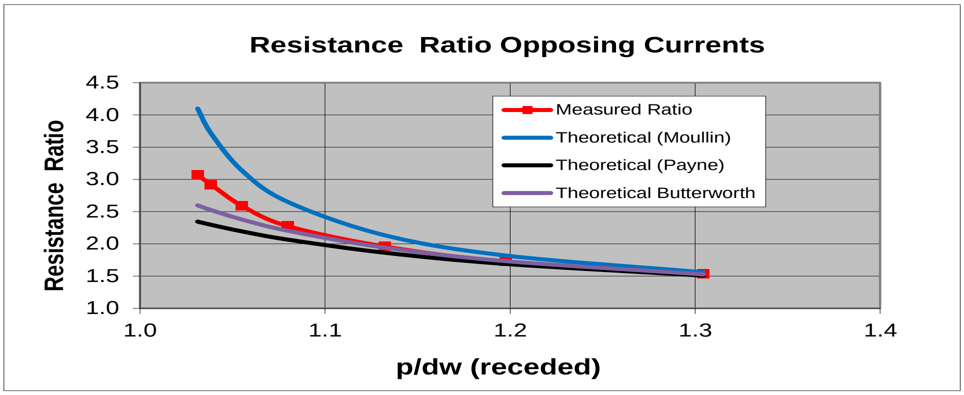

Figure 4.2.1 Resistance Ratio for Opposing Currents

The measurements tend to support Butterworth's equation. However the experimental error is very high and a very small unintentional gap between the wires modifies the calculated values to give greater support to Moullin's equation (see Appendix 3).
So the measurements were inconclusive but nevertheless demonstrated an important point, which is that a p/d_w of less than 1.25 is very unlikely in practice, and so any of the three equations would be suitable. Given that Moulin's equation is the simplest (Equation 4.1.3) this is the most useful of the three.

5. PROXIMITY LOSS IN MULTIPLE PARALLEL WIRES

5.1. Introduction

The theory for the resistance of two parallel wires carrying current in the same direction (Section 3) is extended here to many wires in parallel.

5.2. Theory

In Section 3 the following equation was shown to give a good prediction of the resistance of two parallel wires carrying current in the same direction :

$$R/ R_o = 1 + 1/ (2 x^2 +1) \qquad 5.2.1$$

where $x = (p/d')$
p is the distance between wire centres
$d' = d_w - \delta$
R_o is the resistance of an isolated single wire

If there are a large number of wires, and all are on *one* side of the wire in question, then the above equation becomes :

$$R'/ R_o = 1 + [1/ (2 x_1^2 +1)] + [1/ (2 x_2^2 +1)] + [1/ (2 x_3^2 +1)] + [1/ (2 x_4^2 +1)] \ldots \qquad 5.2.2$$

In general a wire will have a number of wires on *both* sides, say n_1 and n_2, and so the terms in brackets are repeated for the other side, and the resistance ratio of *one* wire is :

$$\mathbf{R /R_o = 1 + [1/ (2 x_1^2 +1)] + [1/ (2 x_2^2 +1)] + [1/ (2 x_3^2 +1)] + [1/ (2 x_4^2 +1)] \ldots\ldots\ldots x_{n1}}$$

$+ [1/(2 x_1^2 +1)] + [1/(2 x_2^2 +1)] + [1/(2 x_3^2 +1)] + [1/(2 x_4^2 +1)] \ldots\ldots\ldots \quad x_{n2}$ 5.2.3

If all these wire are laid side by side and touching, then x will have integer values $x_1= 1$, $x_2= 2$, $x_3= 3$, $x_4 = 4$ ….

The resistance of all the wires in parallel will be the parallel combination of the individual wires :

$$1/R'_t = 1/R'_1 + 1/R'_2 + 1/R'_3 + 1/R'_4 \ldots\ldots\ldots \qquad 5.2.4$$

where R'_1, R'_1 are the normalised resistances of each wire as given by Equation 5.2.3

For instance, for 3 wires the resistance of the individual wires from Equation 5.2.3 is :

middle wire : $R_1 / R_o = 1+ 1/(2+1) +1/(2+1) = 1.67$

outside wire: $R_2 / R_o = 1+ 1/(2+1) +1/(8+1) = 1.44$

outside wire: $R_2 / R_o = 1+ 1/(2+1) +1/(8+1) = 1.44$

The parallel resistance of these is 0.5 R_o, where R_o is the resistance of one isolated wire. Without the proximity loss the parallel resistance of these three wires would be $R_o/3$ and so the proximity effect increases the resistance by the factor 0.5/(1/3) = 1.5.
NB the above assumes that currents induced in one wire do not themselves induce currents into the adjoining wires.

5.3. Experimental Support

In Appendix 5 measurements are given of the resistance of 5 wires, both that of the individual wires and the combined resistance of all five in parallel when connected together at their ends. The latter gave the resistance values in blue below:

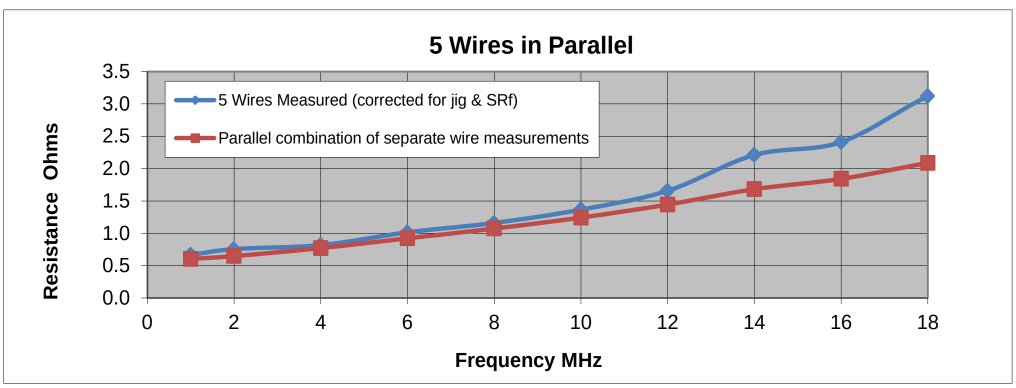

Figure 5.3.1 Resistance of 5 Wires side by side

Also shown (red) is the parallel combination of the *measurements* of the individual wires :

$$R = 1/[1/R_1 +1/R_2+1/R_3+1/R_4+1/R_5] \qquad 5.3.1$$

The correlation is good at the low frequencies, but degrades above 10 MHz probably because the SRF with 5 wires in parallel is slightly different to that of each individual wire, and indeed if it is assumed that the SRF for the 5 in parallel is reduced from 35 to 29 MHz the correlation at high frequencies is the same as at the lower frequencies. As with the previous measurements with the Nichrome wire the SRF could not be accurately measured because the high loss resistance prevented a well-defined resonance.

So these measurements show that the resistance of the 5 wires as a unit is equal to the parallel resistance of the individual wires.

5.4. Multiple Parallel wires

Calculation of Equations 5.2.3 and 5.2.4 have been carried-out for wire groups of 2, 3, 4, 5, 6, 8, 10, 12 and 18 wires, with the following results (in blue) :

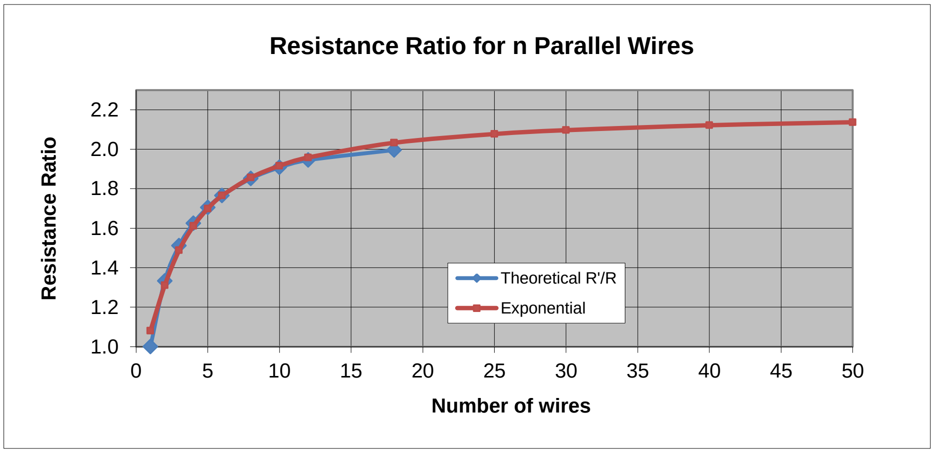

Figure 5.4.1 Resistance Ratio for n Parallel Cylindrical Wires

At very large values of n the curve is asymptotic to 2.27, a value derived in Appendix 8.
Also shown above is the following empirical equation which fits the calculated data to within ±1.6% for n=2 to infinity:

$$R/R_{on} \approx 1 + 1.2 / e^{(2.7/n)} \qquad 5.4.1$$

where n is the number of wires
R_{on} is the parallel high frequency resistance of n wires, assuming no proximity loss.

It is emphasised than R_{on} is the resistance of n parallel wires without proximity effect and at high frequencies so that the skin depth is a small fraction of their diameter.

6. RESISTANCE OF A CONDUCTOR WITH A RECTANGULAR CROSS-SECTION

6.1. Flat sheet Conductor

The previous section showed that with a sheet of wires those at the edges had a lower resistance than those in the centre. Thus for a given potential across the ends a higher current will flow at the edges than at the

centre. The same effect is seen in flat rectangular conductors, and this is then known as current crowding. Equation 5.4.1 is therefore a description of the current crowding across the band of wires and here we explore how this equation might be applied to a thin flat sheet.
Firstly we need to find the equivalent number of wires which can represent a flat sheet. One possibility is shown below where the sheet thickness t is equal to the diameter of the wires d and the width of the sheet is equal to n d, where n is the number of wires.

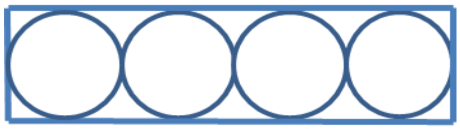

Figure 6.1 Strip Conductor as series of Circular Conductors

In this case the width w is equal to nd and the thickness t is equal to d, so n = w/t. However it is likely that the equivalent flat sheet is smaller in thickness and smaller in width than shown above, and if the cross-sectional areas are the same then the DC resistances will be the same. So $wt = n \pi d^2/4 \approx n \pi t^2/4$, so that :

$$n = (\pi/4)\, w/t \qquad 6.1.1$$

Equation 5.4.1 then becomes :

$$R/R_{os} \approx 1 + 1.2 / e^{2.1\, t/w} \qquad 6.1.2$$

R_{os} is the high frequency resistance of the flat conductor if there was no current crowding. That is, it is the resistance if conduction is assumed to takes place in a thin skin around the periphery, and with constant current density. The above equation thus represents the increase in resistance due at the edges and corners of the strip.

6.2. Rectangular Conductor

It is assumed here that a rectangular conductor at high frequencies can be represented by four of the above flat sheets joined at their edges. So the sum of the four faces (2 faces plus 2 faces) is :

$$\mathbf{R/R_{os} \approx 1 + 1.2 / e^{2.1\, t/w} + 1.2 / e^{2.1\, w/t}} \qquad \mathbf{6.2.1}$$

This equation is plotted below :

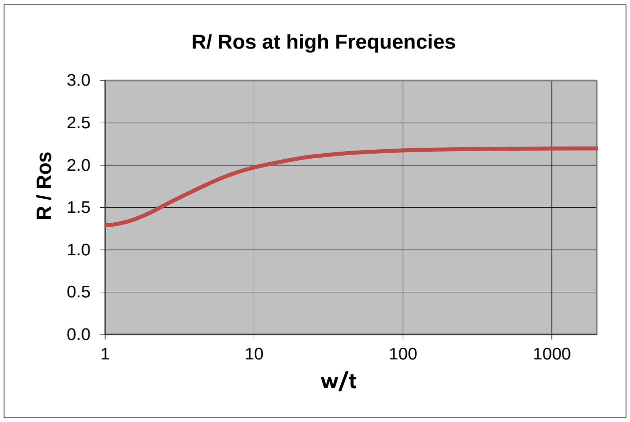

Figure 6.2.1 Strip Conductor as series of Circular Conductors

The above analysis is far from rigorous but it does have some experimental support at large values of w/t (see ref 13).
The author's work on rectangular conductors has expanded considerably to the extent that it justified a separate document and this is given in reference 13.

7. SUMMARY OF EQUATIONS

The resistance of a single isolated circular wire for any value of d/δ is given to a good approximation by :

$$R_o / R_{dc} \approx 1/(1- e^{-x}) \qquad 7.1$$

where $x = 3.9/(d'/\delta) + 7.8/(d'/\delta)^2$
$R_{dc} = \rho\, \ell / A$
d' is the receded diameter of the wire $= d_W - \delta$
d_W **is the wire diameter**
$\delta = [\rho/(\pi f \mu)]^{0.5}$
$\mu = \mu_r\, \mu o$
μ_r **is the material relative permeability**
$\mu o = 4\pi\ 10^{-7}$
ρ = resistivity (ohm-metres)

This has an accuracy of better than ± 3.6% for any value of d/δ.

The resistance of a pair of circular wires with currents in the same direction is given by :

$$R \approx R_o\,[1 + 1/(2\,x^2 + 1)] \qquad 7.2$$

where $x = (p/d')$
p is the distance between wire centres
d' is the receded diameter of the wire $= d_w - \delta$
R_o **is given by Equation 7.1**

The resistance of a pair of circular wires with currents in opposite directions is given by :

$$R \approx R_o / [1- (d'/p)^2]^{0.5} \qquad 7.3$$

The resistance of a n circular wires, insulated or un-insulated, laid side-by-side and connected together at each end is given by :

$$R/R_{on} \approx 1 + 1.2 / e^{(2.7/n)} \qquad 7.4$$

where n is the number of wires
R_{on} **is the parallel high frequency resistance of n wires, assuming no proximity loss.**

Appendix 1 PROXIMITY LOSS IN TWIN WIRES

A1.1. Opposing Currents

In the illustration below current flowing in the first conductor produces a magnetic field in the second conductor, and this induces eddy currents in this second conductor which lead to a power loss. This power must come from the first conductor, since it is responsible for generating the magnetic field, and so *its* resistance increases.

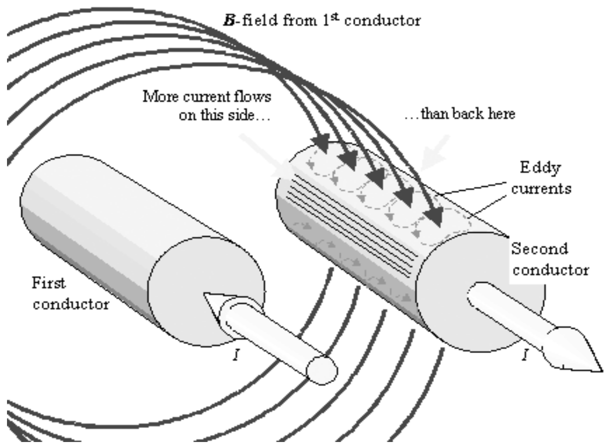

Figure A1.1 Induced Currents in Twin Wires

The magnetic field can be assumed to be generated by a thin filament at the centre of the first conductor carrying a current I. The magnetic field intensity H at a distance r from this filament is :

$$H = I/(2\pi r) \qquad \text{A1.1.1}$$

At high frequencies where the skin depth is small compared with the wire diameter, there will be no flux inside the wire and the circumference will coincide with a line of force of the field.
The power lost is proportional to H^2, and so the power lost in the first conductor of radius a_w, in the absence of any other conductor is:

$$P_0 = K\,H_1^2 = K\,I^2/(2\pi a_w)^2 \qquad \text{w/metre} \qquad \text{A1.1.2}$$

where $K = R_{wall}\,(2\pi a_w)$

The resistance of this conductor is P_0 / I^2 and so :

$$R_0 = K/(2\pi a_w)^2 \quad \text{Ohms} \qquad \text{A1.1.3}$$

and the power lost in the second conductor, at a distance r and due to the current I in the first conductor is :

$$P_r = K\,H_2^2 = K\,I^2/(2\pi r)^2 \qquad \text{w/metre} \qquad \text{A1.1.4}$$

Since this power must come from the first conductor *its* resistance increases by :

$$R_2 = K/(2\pi r)^2 \qquad \text{A1.1.5}$$

So the increase in resistance, as a ratio of the resistance of the first conductor is:

$$R_2/ R_0 = (a_w/r)^2 = (1/r')^2 \qquad \text{A1.1.6}$$

Where r' is r normalised to a_w

Integrating over the distance to the second conductor, gives :

$$R_2/ R_0 = \int_{r_1}^{r_2} (1/ r')^2 \, dr' = \Big[(1/r'_1) - (1/r'_2)\Big]_{r_1}^{r_2} \qquad \text{A1.1.7}$$

The total resistance is therefore :

$$\mathbf{R/R_o = 1 + [(1/r'_1) - (1/r'_2)]} \qquad \textbf{A1.1.8}$$

The limits r'_1 and r'_2 are the distances to the nearest edge of the second conductor and to its farthest edge, measured from the centre of the first conductor, and so $r'_1 = (p/a_w - 1)$ and $r'_2 = (p/a_w + 1)$, when normalised to a_w. These are more conveniently expressed as $r'_1 = (2\,p/d_w - 1)$ and $r'_2 = (2p/d_w + 1)$.
In the analysis above the magnetic field was assumed to be generated by a thin filament at the centre of the first conductor but the effect of the current in the adjacent conductor is to move this filament off-centre, towards the adjacent conductor. The limits of integration r'_1 and r'_2 are therefore reduced, and by an amount depending upon the proximity of the second conductor, p/d_w. For high frequencies where the skin depth is small compared with the wire diameter, Moullin states (ref 6 page 169-170) 'the centres of the lines of force will move from the centres of the wires to the mutually inverse points for both wires'.
The mutually inverse point is sometimes called the image in a circle, and is derived from electrostatics. The diagram below shows a conducting metal cylinder with an external charge –q. By the method of images it can be shown that the field external to the shell is the same as if the shell were non-conducting but had a charge +q inside it. This charge is not centred on the shell but displaced from its centre, towards the external charge by a distance $\Delta = a_w^2 / p$ (Ramo and Whinnery, ref 2 p81)

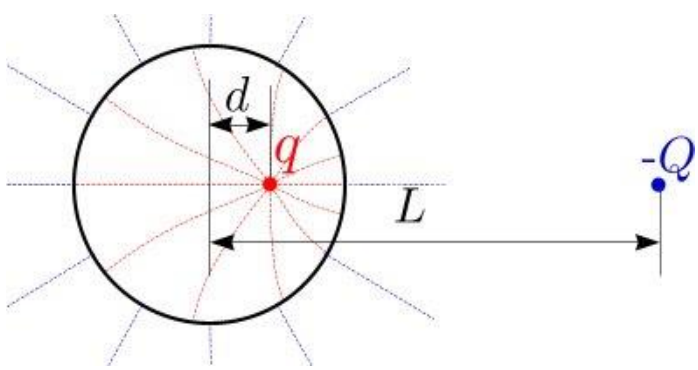

Figure A1.2 Image of a Line Charge in a Cylinder

So in the above analysis the surface of the circular wire is assumed to be coincident with a line of constant intensity in Figure 4.1.1, assuming that the lines are circles. This is approximately true for the inner lines and to the extent that this is true the above theory is valid (Harnwell, ref 9 p33). The magnetic field around current carrying wires is exactly the same as the electric field around charges, and so the theory can be applied to this problem.

Since we are normalising all dimensions to a_w this displacement from the centre becomes $\Delta' = a_w / p = 0.5 d_w/p$, and Equation A 8.1.8 becomes for *opposing currents :*

$$R/R_o \approx 1 + [(1/r'_1) - (1/r'_2)] \quad \text{A1.1.9}$$
$$\text{where} \quad r'_1 = (2p/d_w - 0.5\, d_w/p - 1)$$
$$r'_2 = (2p/d_w - 0.5\, d_w/p + 1)$$

A1.2. Currents in Same Direction

In Figure 8.1.1 the currents are shown in opposite directions, and the current density is higher on the inner surface of each conductor, but when currents are in the same direction the current density is higher on the *outer* surface. The image therefore moves out from the centre and Equation 8.1.9 becomes :

$$R/R_o \approx 1 + [(1/r'_1) - (1/r'_2)] \quad \text{A1.2.1}$$

$$\text{where} \quad r'_1 = (2p/d_w + 0.5\, d_w/p - 1)$$
$$r'_2 = (2p/d_w + 0.5\, d_w/p + 1)$$

NB recession of the current is not included in the above equations, but has been included in Equations 3.2.1 and 4.1.1

Appendix 2 RESISTANCE MEASUREMENTS, CURRENTS IN SAME DIRECTION

A2.1. Experimental Issues

For the measurement of the resistance of wires with currents in the same direction copper magnet wire would be very suitable because its purity is well controlled. However its low resistance causes measurement problems, as given below.

If the tests are to extend to high frequencies (since the equations are for high frequencies) then the SRF (Appendix 7) must be sufficiently above the maximum measurement frequency. Also the wire diameter must be large compared with the skin depth at the measurement frequencies. Together these imply a wire with a short length and a large diameter, but then the resistance will be very low and this cannot be measured with sufficient accuracy. The resistance could be increased by using a smaller diameter wire, but then the measurement frequency would have to increase to maintain the skin depth ratio. The wire length would then have to be reduced to increase the SRF, negating the advantage of the thinner wire. Another disadvantage of thin wire is that it has a higher thickness of insulation in proportion to its diameter, preventing close spacing of the wires.

Overall there is a compromise to be made and it was decided to use 4 mm diameter copper wire, with a length of 6 m. The SRF when folded back on itself (Appendix 7) was around 17 MHz and so measurements were made from 0.5 to 6 MHz. The lowest frequency is determined by the skin depth, which needs to be small compared with the wire diameter (25% of the wire dia, at 0.5 MHz), and also by the measurement errors, which will increase because of the low resistance at low frequencies. The highest frequency is determined by the increase in resistance due to the SRF (a factor of 1.3 at 6 MHz) and the errors in correcting for this, although in compensation the higher resistance at high frequencies reduces the measurement error.

Two measurements were made, firstly of the ac resistance of a single wire and then of two wires arranged side by side. This was achieved by twisting the two together, with 200 turns, and then giving the whole length a slight stretch to minimise untwisting when the tension was removed. The two wires were soldered together at each end, the resistance measured and compared with that of the single wire. To minimise the SRF the single wire was folded-back on itself with spacing of only around 30 mm, and the same was done with the twisted pair. It was hoped that the SRF would be the same, since then no correction would need to be made for the SRF, but it was found that they gave 16.7 and 18.1 MHz respectively (measured as the frequency where the phase of Z_{in} went through zeo), giving a correction of 1.32 and 1.26 respectively at 6 MHz. Despite the long length of wire the resistance values were low, ranging from 0.8 to 2.9 Ω for the twisted pair, and given that the objective was to measure a change of around 20% this equals a change of only 160 mΩ. If this is to be measured to say 10 % the measurement error must be less than 16 mΩ, and

this required careful calibration. In particular it was important that changes in the resistance of the *connectors* used to interface the wire to the VNA were eliminated because contact resistance at each interface is around 4 mΩ for the SMA connectors used. It was therefore not possible to use the normal calibration procedure of a short-circuit by replacing the wire and its connector with an SMA short circuit. Instead with the wire soldered to the connector and this connected to the VNA a short-circuit was placed across the *wire* and the measured resistance subtracted for the subsequent measurements, and these need to be done immediately afterwards to avoid errors due to temperature changes. This 'zero resistance' measurement had to be repeated whenever the wire or the connectors were changed, since this measurement could change as much as 30 mΩ.
In principle the measurement of the twisted pair could be made with only one wire connected to the measurement equipment and the other left free, and this would have the advantage that the resistance values would be double that when both wires were connected. This was tried initially but it was found that, in addition to the expected SRF, there was a low Q resonance at a lower frequency. The cause of this was not investigated but it is likely to be due to a transmission-line mode between the two wires, whose phase velocity was slowed by the enamel insulation.

A2.2. **Comparison of Experiment with Theory**

The measurements gave the following, (in red):

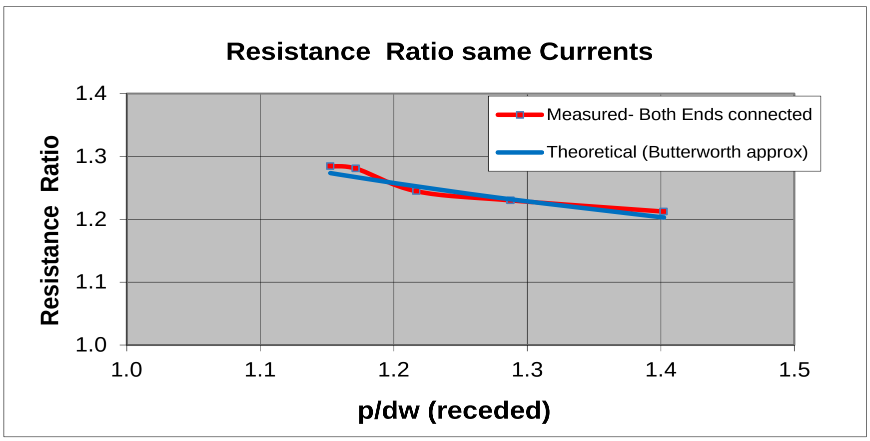

Figure A2.2.1 Resistance Ratio for Currents in Same Direction : Measured and Theoretical

Also shown (blue) are the theoretical values given by Equation 3.2.1, and the correlation is seen to be very good and within ±1%.
The above graph shows a range of wire spacings p/d_w, but clearly the tests gave only one *physical* value of p/d_w, and this was 1.08 set by the thickness of the enamel insulation (measured). However at the frequencies used here, the effective wire diameter reduces by the skin depth (see Section 2.4), and this increases p/d, and by an amount depending upon the frequency.

A2.3. **The Effect of Twisting**

In the above experiment the two wires were held close by twisting them together. It was assumed that the resistance would be unaffected by this mild twisting, however Woodruff (ref 7, p67) says 'For stranded conductors in which the separate strands lie parallel to the axis (without spiralling) the skin-effect resistance ratio has been shown by tests to be the same, within experimental error, as for solid wires having the same cross-sectional area of metal (not the same outside diameter). If the strands are wound spirally the same condition holds very closely at low frequencies such as are used in power work, but for frequencies above about 1200 cps on the cable tested the skin-effect was found to increase more rapidly than solid wires of the same area'. Unfortunately he does not give a specific reference for this work, but some recent measurements are given in reference 14.

In the author's tests above the single wires were 6000 mm long and given approximately 200 twists, and then the whole length given a slight stretch to avoid untwisting when the tension was removed. The stretched length was 6005 m, so the length of lay was 6005/200 ≈ 30 mm. To test the effect of twisting, a further 400 twists were made, again with a slight stretching to give an overall length of 6090 so the length of lay was approximately 10 mm.
The resistance with both 200 twists and 600 twists was measured and were within 3%, ie within the measurement repeatability (in fact, ironically the larger number of twists gave the *lower* resistance).

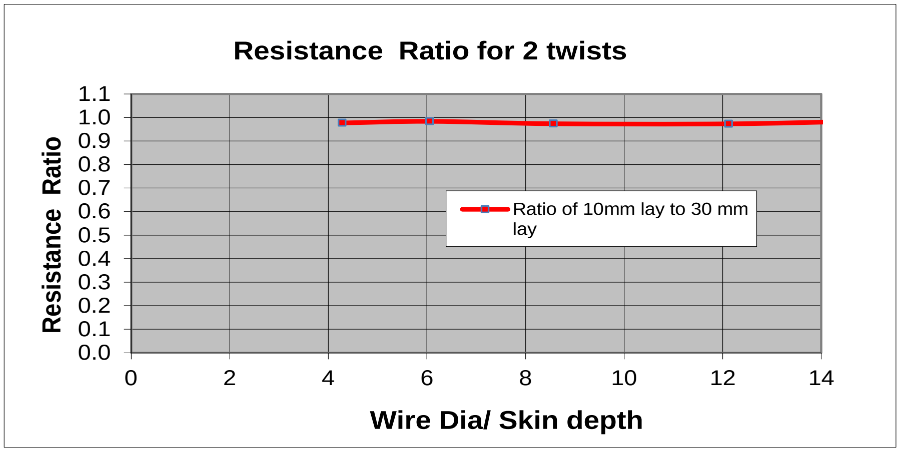

Figure A2.3.1 Measured Resistance Ratio for two Values of Twist

So it was concluded that the twisting did not have any measureable effect on the resistance, at least when the skin depth was small compared with the diameter.

Appendix 3 RESISTANCE MEASUREMENTS, OPPOSING CURRENTS

A3.1. Introduction

The three equations given in Section 4.1, for the resistance ratio when the currents are opposing, are within 5% of one another for p/d_w greater than 1.25, and so to distinguish between them the experiment must have p/d_w ratios much less than this. However such close spacing is very difficult to achieve using commercially available magnet wire since the insulation thickness is about 5% of the wire diameter, and so the *minimum* physical spacing of p/d_w is around 1.1. In addition, the electrical p/d_w is greater than this due to the recession of the current into the wire by the skin depth (Section 3.3). To minimise this recession the measurements should be done at a high frequency, but then the measurement frequency may become close to the SRF leading to potentially large errors, even after correction.
Two experiments were carried out, one using conventional insulated magnet wire (accepting the limitation caused by the thickness of the insulation), and the second using wire which had been stripped of its insulation in order to reduce the wire spacing. These are described below.

A3.2. Experiment with Insulated Magnet Wire

With conventional enamel coated magnet wire it is difficult to obtain an electrical p/d_w less than 1.1, and at this value the difference between the theories is only 13%. This meant that each source of error needed to be kept to less than 1% if possible.
Great efforts were made to minimise errors (see later), and the following ratio was measured :

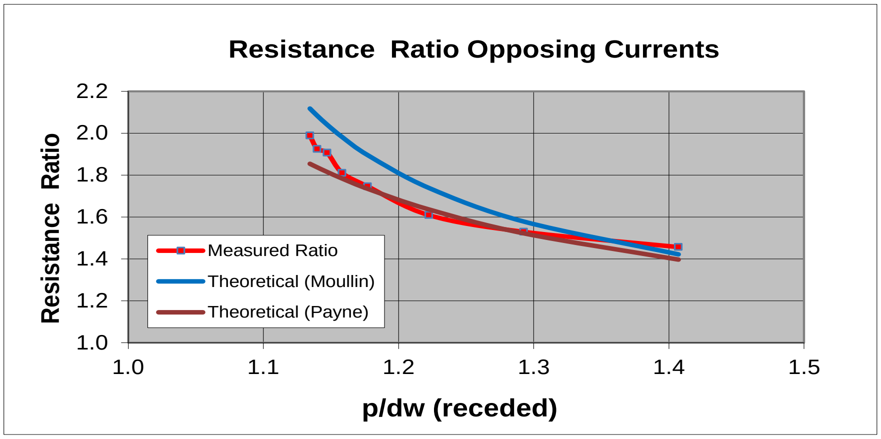

Figure A 3.2.1 Resistance Ratio for Opposing Currents, Measured and Theoretical

Also shown are ratios calculated from Equation 4.1.1 (Payne) and Equation 4.1.3 (Moullin). The measurements lie between the two equations, and so are inconclusive.
The above graph shows a range of wire spacings p/d_w, but clearly the tests gave only one *physical* value of p/d_w, and this was 1.08 set by the thickness of the enamel insulation (this was measured). However at the frequencies used here, the effective wire diameter reduces by the skin depth (see Section 3.3), and this increases p/d_w, and by an amount depending upon the frequency.

Details of experiment :
The above is the ratio of the measured resistance of a single wire of length 2 meters and that of a pair of wires each 1 meter long, twisted together, and joined at one end. The wire diameter was 0.4 mm. Each wire was soldered to a single-ended SMA connector which was then mated with the VNA. With the single wire, it clearly has to be formed into a loop for connection to the test equipment, but this was found to be problematic because the resultant high inductance reduced the measurement accuracy, and also this loop did not have a well-defined SRF, making correction for this impossible. The solution was to fold the wire back on itself with a centre to centre spacing between the wires of about 2 mm, and this close spacing will increase its resistance by 2.2% according to Equation 4.1.3. The measurements were corrected for this.
Correction for the self-resonance is significant, and the SRF was measured as the frequency where the phase of the input impedance passed through zero. In applying Equation 14.1, previous experience with transmission lines (as against coils) showed that best accuracy was achieved in the correction if the SRF was assumed to be 85% of the measured value. The measured SRF for the single wire was 59.7 MHz and for the twin 50.3 MHz.

The measurement procedure was designed to minimise errors as far as possible, and in this respect there are two types of measurement: a) the physical measurements of wire diameter and spacing between conductors, and b) the electrical measurements.
Taking the physical, the wire diameter was measured as 0.4025 mm with a micro-meter, after the insulation had been removed by heating in a flame. The diameter over the insulation was measured as 0.435 mm. The wires were twisted together and stretched slightly to prevent the twists unwinding. It was assumed that the physical spacing was equal to the thickness of the insulation, however there could have been some gaps which would have increased the spacing between the wires. Alternatively the stretching could have deformed the wire diameter, thereby reducing the spacing.
The resistance values to be measured are between 0.5 and 3.5 Ω and the contact resistance in the (SMA) connectors to the VNA was found to be significant, and to reduce the errors here the connectors were tightened with a spanner. Also the short circuit calibration procedure was as described in Appendix 2.

A3.3. Experiment 2 : Stripped wire

The above experiment was unable to distinguish between the theories because at the achievable spacing the difference between the theories was too small. A smaller spacing was therefore required, and this was

achieved by stripping the enamel insulation from 0.4 mm diameter copper wire by heating it in a gas flame for a few seconds until the wire was red hot. The temperature was probably around 1000^0 C, and at this high temperature an insulating oxide layer was formed. The thickness of this layer is difficult to determine but is likely to be of the order of 20 µm.
Two such wires each 0.51 m long were twisted together and stretched slightly to prevent unwinding. To check the insulation the resistance between these wires was measured and gave a value of 0.6 MΩ. The ac resistance of these wires was measured as go and return, and compared with the *theoretical* resistance of a single wire of length 1.02 m long, with the following results. During the measurements a small tension was maintained by hand to ensure close winding :

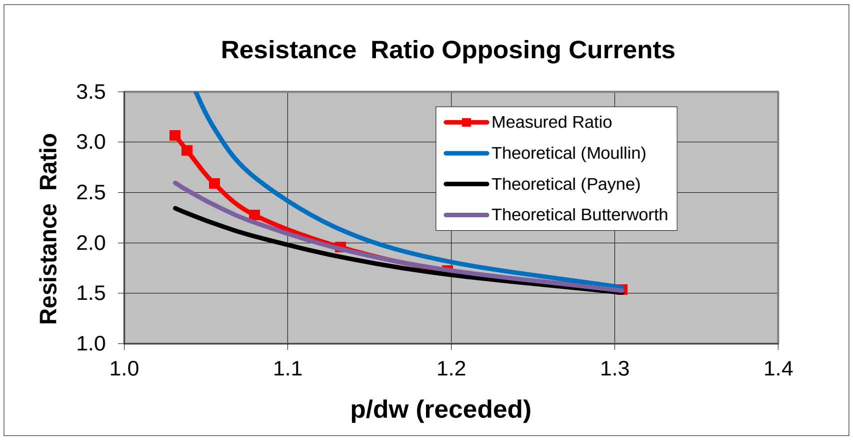

Figure A3.3.1 Resistance Ratio for Opposing Currents, Measured and Theoretical

The above graph shows a range of wire spacings p/d_w, but clearly the tests gave only one *physical* value of p/d_w, and this was 1.0001, assuming the gap between the wires was due to the oxide layer only. However at the frequencies used here, the effective wire diameter reduces by the skin depth (see Section 2.4), and this increases p/d_w, and by an amount depending upon the frequency.
The resistance ratio given at low values of p/d_w is measured at high frequencies (up to 30 MHz) and so is highly dependent upon the correction given to the measured values by self-resonance. For this the SRF was measured as 92.5 MHz, being the frequency where the phase of the input impedance passed through zero. Previous experience has shown that a more accurate correction is achieved if the SRF is assumed to be 85% of the measured value and this was used above.
The above shows that the measured values give greatest support to Butterworth's theory, Equation 4.1.2. However, the experimental error is very large, and for instance if there had been an unintentional gap between the wires of only 0.01 mm this would significantly change the p/d_w ratio to give the following :

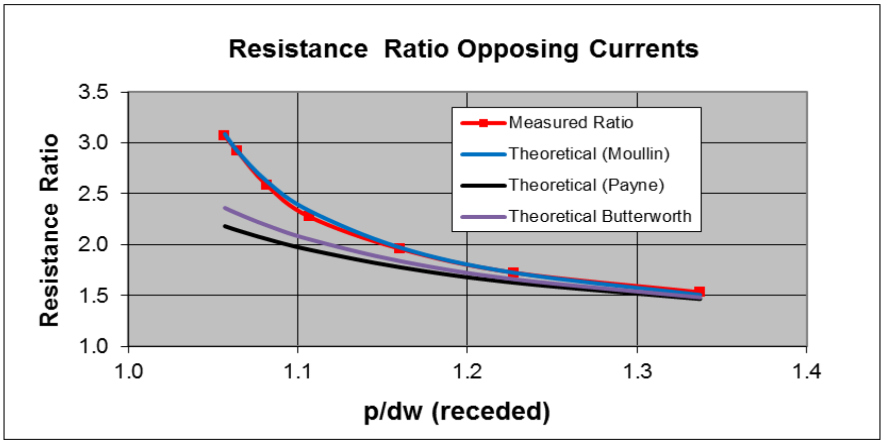

Figure A3.3.2 Resistance Ratio for Opposing Currents, Measured and Theoretical

This gives greater support to the Moullin equation especially since close examination showed some evidence of a very small gap in some parts of the winding.
One aspect not considered above is displacement current, due to the capacitance between wires and this was measured at 120 pf, giving a reactance at 30 MHz (the highest measurement frequency) of –j42 Ω. However the correction due to the SRF already accounts for the distributed L and C so no further correction is needed. Welsby's equation for the rise in resistance due to the SRF (Appendix 7) assumes that $1/Q^2$ is small compared with unity, and that is true here (just) where the measured Q was around 4.4 at the high frequencies (30 MHz) where the correction is most significant.

Appendix 4 SKIN DEPTH OF NICHROME WIRE

The ac resistance of a straight isolated wire can be expected to conform to the following equation:

$$R_{ac} = R_{dc}\, 0.25\ (d_w / \delta)^2\ /(d_w/\delta - 1)]\ /\ [1- (f/f_{srf})^2]^2 \qquad \textbf{A4.1}$$

The first part of the equation was derived in Section 2.3, and the second part is due to self-resonance, Appendix 7. With copper wire this equation gives accurate predictions but this was not so for Nichrome wire, as shown below for a wire of diameter 0.72 mm and the length 68 mm. This short length gave a high SRF of around 400 MHz so any uncertainty here would have minimum effect on the measurements at frequencies up to the maximum of 30 MHz :

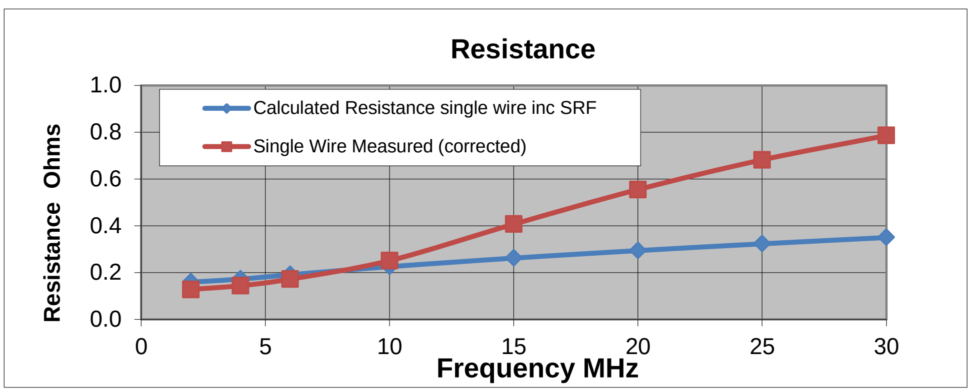

Figure A4.1 Resistance of Nichrome wire, Measured and Calculated

In red is the measured resistance and in blue the resistance calculated from Equation 11.1, assuming the skin depth is given by a resistivity of 1.08 10^{-6} Ωm (measured at dc) and μ_r=1.
It is seen that the measured resistance at high frequencies is considerably larger than predicted, and it was suspected that the Nichrome wire had a thin outer layer of material having either a higher resistivity and/or higher permeability than the bulk of the wire. This would not affect low frequencies very much because the skin depth would be much deeper than the layer, but the current at high frequencies may flow mainly in the thin layer. Assuming that there is a well-defined layer, rather than a gradual change of resistivity and/or permeability, the theory can be made to match the measurements if the layer is 0.015 mm thick with a permeability of 30. However there are many combinations which give the same overall resistance, for instance μ_r = 500 and t = 0.0035 mm. This high value of μ_r would correspond to a layer of iron on the surface, although this is extremely unlikely since the wire does not rust. However it is known that Nichrome wire often has a small iron content, and indeed the Nichrome wire used *was* attracted to a permanent magnet.

Appendix 5 MEASUREMENTS ON 5 PARALLEL WIRES

A5.1. Introduction

The experiment used 5 wires in parallel and the ac resistance of each was measured in the presence of the others, these being open circuit (ie not connected). Since the structure is symmetrical about the centre wire only 3 wires need be measured : one outside wire, the middle wire, and the one between the two. Also measured was the ac resistance of the 5 wires in parallel when connected together at their ends. Note that the wires must be insulated from one another for this experiment, or current will flow from one to the others via contact the resistance.

Five Nichrome wires of 0.72 mm dia were cut to 1meter length, and stretched to make them straight, and their stretched length was 1.01 m. To connect to the measurement equipment a return conductor of 2 mm copper wire was spaced by 25 mm from the 5 wires. The whole structure was mounted on a wooded board, with the 5 wires held in place by 7 plastic clamps screwed to the wood, and the copper wire fixed to the wood with adhesive tape. This structure ensured that the 5 wires were always the same distance apart, and also the same distance from the copper return conductor, and this ensured that the SRF was always the same - this was important since the SRF was around 35 MHz, not much higher than the maximum measurement frequency of 15 MHz. The proximity loss between the 5 wires and the return copper conductor was calculated from Equation 4.1.3 as 1.003 assuming the 5 wires were equivalent to a single 2 mm dia wire (the same as the copper return conductor). This proximity factor is so small that the added complexity of assessing the proximity loss to the 5 wires was not necessary.
Alternate wires were coated in a thin layer of varnish by brushing-on and then hanging the wire vertically so that the excess would drain off. The estimated thickness of the varnish was 0.02 mm.

A5.2. Results for Individual wires

The measured resistance of 3 of the wires in the group of 5 are given below. The resistance of the other two wires are assumed the same, since there was symmetry about the centre conductor. For comparison the resistance of a single isolated conductor was also measured.

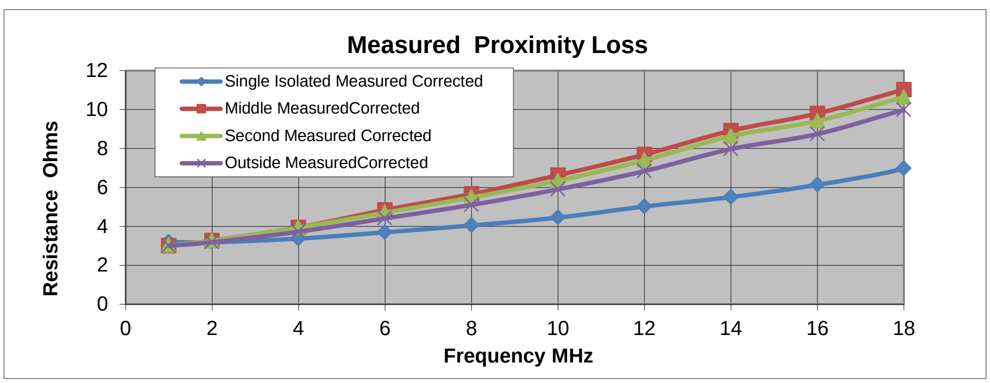

Figure A5.2.1 Resistance of Wires in a Group of Five

All conductors showed a higher resistance than the single conductor, with the central conductor showing the highest resistance, as expected.
At the lowest frequency all conductors showed essentially the same resistance as the single isolated wire, and this is because the skin depth at these frequencies was about equal to the wire diameter, so there was almost complete penetration by the flux.
The measured results have been corrected for an SRF assuming this is 35 MHz, but this was not well defined because the phase did not go through zero at any frequency but an initial change in phase seemed to be heading towards zero at this frequency. However, since the aim here was to measure the relative resistance of each wire an error in the SRF makes no difference.

A5.3. Results for all 5 Wires in Parallel

The 5 wires in the above experiment were connected in parallel at each end and the ac resistance measured as follows (blue):

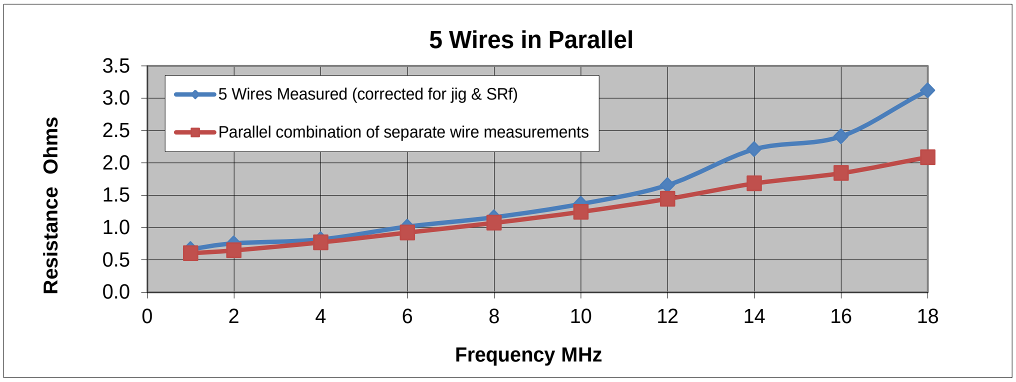

Figure A5.3.1 Resistance of Wires in a Group of Five

Also shown (red) is the parallel combination of the measurements of the individual wires :

$$R = 1/[1/R_1 + 1/R_2 + 1/R_3 + 1/R_4 + 1/R_5] \qquad \text{A5.3.1}$$

The correlation is good at the lower frequencies although there is a consistent 10% difference between the two, and the most likely cause is measurement error, because an error of only about 0.1Ω would account for this difference.

The correlation is not good above 10 MHz and this is probably because the SRF of the 5 wires in parallel is slightly lower than that of each individual wire, and indeed if it assumed to be 29 MHz rather than 35 Mhz the correlation at high frequencies is the same as at the lower frequencies.

The important conclusion from this is that the resistance of the 5 wires as a unit is equal to the parallel resistance of the individual wires (all resistances measured).

A5.4. Testing the Equations for Individual Wires

The measurements in Figure 12.2.1 give the resistance of each wire in the presence of the others, and the ratio of each resistance to that of a single isolated wire (also measured) is given by the equations in Section 5.2. The measurements can therefore be used to test these equations.

Middle Conductor

Taking the middle conductor of 5 conductors, the resistance will be :

$$R'_5 = 1 + R_1 + R_2 + R_3 + R_4 \qquad \text{A5.4.1}$$

Where R_1, R_2, R_3 and R_4 are the added resistance due to each of the other 4 wires, given by Equation 5.2.1 repeated below

$$R_n = R_o\,[1 + 1/(2x^2 + 1)] \qquad \text{A5.4.2}$$

$$\text{where} \quad x = (p/d_w)$$

If x_1 is the value of $(p/d_w)^2$ for the two adjacent conductors, and x_2 the value of $(p/d_w)^2$ for the two outside conductors, the resistance ratio can be anticipated to be :

$$R'_5 = 1 + [1/(2x_1^2 + 1)] + [1/(2x_1^2 + 1)] + [1/(2x_2^2 + 1)] + [1/(2x_2^2 + 1)] \qquad \text{A5.4.3}$$

$$= 1 + [2/(2 x_1^2 +1)] + [2/ (2 x_2^2 +1)] \quad \text{A5.4.4}$$

To compare this equation with the *measurements* on the middle wire we need the values of x_1 and x_2. If the wires were touching with no insulation the values would be 1 and 2 respectively. However, in practice there is insulation thickness and also there was a slight gap between the insulated wires. Measurements give an average width of the 5 wires of 4.2 mm, 0.6 mm greater than if the bare wires were touching. We can assume therefore that the gap between wires was 0.15 mm so that $x_1 = (0.72 +0.15)/d_w'$ and $x_1 =2 (0.72 +0.15)/d_w'$, where d_w' is the diameter of the receded current, $d_w' = d_w - \delta$. Putting this into Equation A5.4.4 (with R_o equal to the measured resistance of the isolated wire) gives (red) :

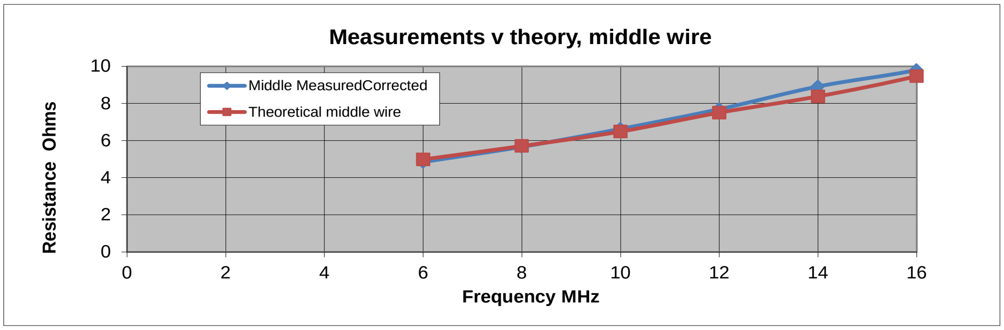

Figure 5.4.1 Resistance of Middle Wire of Five Wires

This shows good correlation with the measured resistance of the middle wire (blue) generally within ±5%.

Outside Conductor

For the outside wire the resistance ratio is :

$$R'_5 = 1 + [1/ (2 x_1^2 +1)] + [1/ (2 x_2^2 +1)] + [1/ (2 x_3^2 +1)] + [1/ (2 x_4^2 +1)] \quad \text{A5.4.5}$$

This gives :

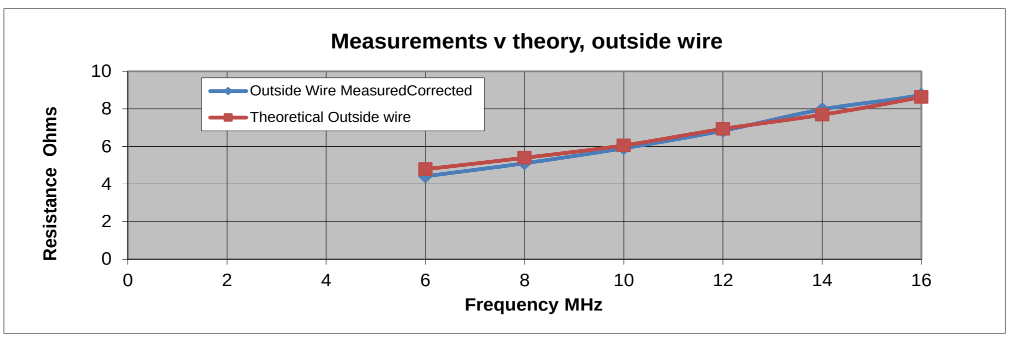

Figure A5.4.2 Resistance of Outside Wire of Five Wires

The measurements again support the theory.

Appendix 6 THE AC RESISTANCE OF ROUND CONDUCTORS

The following derives an empirical equation which gives the ac resistance of round conductors for all values of d/δ.

Initially we note from Section 2.3 that the limiting value of R_{ac} / R_{dc} at large values of d/δ is $(d_w/\delta +1)/4$, so that the slope is $d_w/\delta /4$. In addition at small values of d/δ the value is unity. An empirical equation which satisfies this is :

$$R_o / R_{dc} = 1/(1- e^{-x}) \qquad \text{A6.1}$$

$$\text{Where } x= 4/(d/\delta)$$

[This equation is asymptotic to a slope of $d_w/\delta /4$ at large values of d_w/δ as required since $e^x \approx 1+x$ for small values of x].

This is plotted below along with the accurate curve from Terman's tabulation :

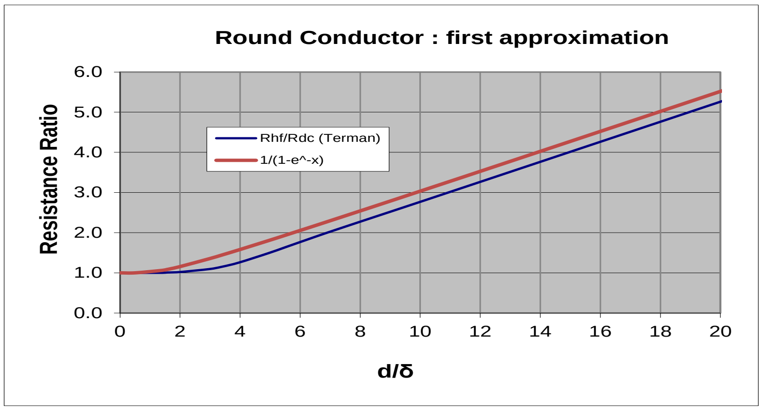

Figure A6.1 Round Conductor : first approximation

The above equation is correct at small values of d_w/δ but is in error by a constant 0.25 at high values of d_w/δ. This can be largely eliminated by an empirical modification to x :

$$R_o / R_{dc} = 1/(1- e^{-x}) \qquad \text{A6.2}$$

$$\text{where } x= 3.9/(d/\delta) + 7.8/(d/\delta)^2$$

This has an accuracy of better than ± 3.6% for any value of d/δ. This equation is plotted in Section 2.4.1.

Appendix 7 THE SELF RESONANT FREQUENCY (SRF)

In experiments a major complicating factor is that all coils and transmission-lines show a self-resonant frequency (SRF), and as this frequency is approached the inductance and resistance increase while the Q decreases. This effect is covered in detail by the author in reference 8, but for the purposes here the following equation is sufficient :

$$\mathbf{R = R_m \, [\, 1- (f / f_{srf})^2]^2} \qquad \textbf{A7.1}$$

In this equation R is the value of resistance which would obtain in the absence of the resonance and R_m is the value that would be *measured* at a frequency f, for a self-resonant frequency of f_{srf}.

These equations are valid if the Q is high enough for $(1+1/Q^2)$ to be taken as unity ie for Q >4. The accuracy of this equation reduces as the SRF is approached and generally the measurement frequency should not be greater than 0.3 f_{srf} if the equation is to give a reliable correction.

If measurements are to be made on a straight wire it has to be formed into a loop, and this will also show a resonant frequency. Equation 2.4.1 then becomes :

$$R_{om} \approx R_{dc} / [\{ 1- (f / f_{srf})^2\}^2 \{1- e^{-x} \}] \qquad \textbf{A7.2}$$

where $x= 3.9/(d/\delta) + 7.8/(d/\delta)^2$

R_{om} **is the measured resistance**

If the loop is large the SRF will be low, giving a large correction along with its uncertainties. To maximise the SRF the straight wire should be folded back on itself to form a shorted open-wire transmission line, but the spacing should not be too small or the proximity loss may become significant (see Section 4). In applying the above equations previous experience with transmission lines (as against coils) showed that best accuracy was achieved if the SRF was assumed to be 85% of the measured value.

Although the SRF can be calculated, it is often best measured because even a very small stray capacitance in the test jig can significantly reduce its value. Clearly the measurement must be done in exactly the same jig as that used for the resistance measurements.

Appendix 8 SERIES LIMIT

Equation 5.2.3 gives the resistance of n wires as :

$$R /R_o = 1 + 2 \ [1/ (2 n_1^2 +1)] + [1/ (2 n_2^2 +1)] + [1/ (2 n_3^2 +1)] + [1/ (2 n_4^2 +1)] \ldots\ldots\ldots \text{A8.1}$$

Where n_1, n_2 .. are integers.

This is a convergent series and it is useful to determine the limit for an infinite number of wires. Ref 11 gives the following equality :

$$\sum_{n=1}^{\infty} y/(n^2+y^2) = -1/(2y) + (\pi/2) \coth (\pi y) \qquad \text{A8.2}$$

Now $1/(2 n^2 +1) = (1/2)/\{ n^2 +(1/2)\}$, so $y = 1/\sqrt{2}$ in Equation 15.2. Thus :

$$\sum_{n=1}^{\infty} [1/(2 n^2 +1)] = 1/(\sqrt{2})[-\sqrt{2}/2 + (\pi/2) \coth (\pi y) \qquad = 0.637 \qquad \text{A8.3}$$

So Equation A8.1 becomes in the limit i.e. a strip with an infinite number of wires ($n=\infty$) :

$$R /R_{on} = 1 + 2 (0.637) = 1.274 \qquad \textbf{A8.4}$$

Where R_{on} **is the parallel resistance of n wires, assuming the current is the same in each.**

REFERENCES

1. WHEELER H A : 'Formulas for the Skin Effect', Proc. IRE, September 1942 p412-424.
2. RAMO S & WHINNERY J R : 'Fields and Waves in Modern Radio' Second Edition, 1962, John Wiley & Sons.
3. TERMAN F E : 'Radio Engineer's Handbook', First Edition 1943, McGraw-Hill Book Company.
4. BUTTERWORTH S : 'Eddy-Current Losses in Cylindrical Conductors, with Special Application to the Alternating Current Resistance of Short Coils', Proceedings of the Royal Society, June 1921. http://rsta.royalsocietypublishing.org/content/222/594-604/57.full.pdf+html
5. BUTTERWORTH S : 'On the Alternating Current Resistance of Solenoid Coils', Proceedings of the Royal Society, Vol 107, P693-715. 1925.
6. MOULLIN E B :'Radio Frequency Measurements', Second Edition, 1931, Charles Griffin & Company Ltd.
7. WOODRUFF L F : 'Principles of Electric Power Transmission', Second Edition, 1938, John Wiley & Sons Inc.
8. PAYNE A N : 'Self-Resonance in Single Layer Coils', http://g3rbj.co.uk/
9. HARNWELL G P : 'Principles of Electricity and Electromagnetism', Second Edition, 1949, McGraw-Hill Book Co Inc.
10. SPREEN J H : 'A Circuit Approach to Teaching Skin Effect' http://ilin.asee.org/Conference2007program/Papers/Conference%20Papers/Session%201A/Spreen.pdf
11. http://en.wikipedia.org/wiki/List_of_mathematical_series#cite_note-5
12. PAYNE A N : 'Skin Effect : Electromagnetic wave or Diffusion ?' http://g3rbj.co.uk/
13. PAYNE A N : 'The AC Resistance of Rectangular Conductors' http://g3rbj.co.uk/
14. SCHRODER G, KAUMANNS J : ' Advanced Measurement of ac Resistance on Skin-effect Reduced Large Conductor Power Cables' : 8th International Conference on Insulated Power Cables, A.8.2, June 2011. http://www.suedkabel.de/cms/upload/pdf/jicable/advanced_measurement_of_AC_resistance_on_skin-effect_reduced_large_conductor_power_cables.pdf

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Issue 2 : March 2015 : Series Limit determined accurately in Appendix 8, and incorporated into Equations 5.4.1, 6.2.3, 6.3.3, 7.1.4, 7.1.5, and 7.1.6.

Issue 3 : June 2016 : Resistance of a strip conductor totally revised (Section 6).

Issue 4 : June 2017 : Empirical equation for round conductor added (Sect 2.3). Appendix format improved

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