THE EFFECT OF DIELECTRIC INSIDE AN INDUCTANCE COIL

Inductance coils are often wound around an insulating former which then provides mechanical support. The magnetic field is not affected by this former and so the low frequency inductance is not changed. However the inductance of all coils increases with frequency due to self-resonance, and the presence of the insulating former will increase this change of inductance because it lowers the self-resonant frequency (SRF). Losses in the former will reduce the Q of the coil, but interestingly it is found that the biggest reduction in Q is due to the reduction in the SRF. So a material with a low dielectric constant is preferable to one with a high dielectric constant, even if the loss factor of the low dielectric material is higher.

1. INTRODUCTION

Inductance coils are often wound around an insulating ‘former’ which then provides mechanical support. The magnetic field around the inductor is not affected by this and so the low frequency inductance is not changed. However the inductance of all coils increases with frequency due to self-resonance, and the presence of the insulating former will increase this change of inductance because it lowers the self-resonant frequency (SRF).

It is shown here that the effective dielectric constant of the former material is much less than its actual dielectric constant because the electric field inside the coil is very small, and is dependent upon the factor $(\pi d_c)^2 / (p \lambda)$ where $d_c$ is the coil diameter, $p$ the pitch of the winding, and $\lambda$ the wavelength. Also the electric field is strongest close to the wire and so it is here that the dielectric has most effect.

Losses in the former will reduce the Q of the coil, but interestingly it is found that the biggest reduction in Q is due to the reduction in the SRF, so that in practice a lossy material with a low dielectric constant can be preferable to a low loss material with a high dielectric constant.

Key equations are highlighted in red, and a summary of these is given in Section 6, with a practical example in Section 7.

2. THE EFFECTS OF DIELECTRIC INSIDE A COIL

2.1. Introduction

The permittivity of a material is given by the product $\varepsilon_r \varepsilon_n$, where $\varepsilon_n$ is the permittivity of free space (=8.854 pf/metre) and $\varepsilon_r$ is the relative permittivity of the material, also called the dielectric constant of the material.

The dielectric constant has real and imaginary components as follows:

$$\varepsilon_r = \varepsilon' - j \varepsilon''$$

2.1.1

$\varepsilon'$ is the normally quoted dielectric constant and $\varepsilon''$ is the loss component. Often $\varepsilon''$ is not given in tables of constants, and instead the ‘dissipation factor’ $\tan \delta$ is given and this is equal to $\varepsilon'' / \varepsilon'$. In RF work it is more convenient to express loss in terms of Q, and this is equal to:

$$Q_{di} = 1 / \tan \delta = \varepsilon' / \varepsilon''$$

2.1.2

The dielectric constant and $Q_{di}$ for a number of materials are given in the table below at 1 MHz and 20°C (ref 12):
2.2. Increased Inductance

The inductance of all coils increases with frequency due to self-resonance (Payne ref 1), and the presence of the insulating former will increase this change of inductance. An example of the frequency change with frequency is shown below:

The effective dielectric constant is much lower than these values, and its calculation is considered in the Sections 3 onwards.

In the following paragraphs it is assumed that the effective dielectric constant $\varepsilon'_{\text{eff}}$ is known.

### Table 2.2.1

<table>
<thead>
<tr>
<th>Material</th>
<th>$\varepsilon'$</th>
<th>$Q_{\text{hi}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wood (Scots Pine)</td>
<td>8.2</td>
<td>17</td>
</tr>
<tr>
<td>PVC-U</td>
<td>2.7-3.1</td>
<td>59 - 167</td>
</tr>
<tr>
<td>Acrylic (Perspex)</td>
<td>3.5-3.5</td>
<td>33 – 50</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>2.56</td>
<td>1400</td>
</tr>
<tr>
<td>PTFE</td>
<td>2 - 2.1</td>
<td>$\geq$ 5000</td>
</tr>
</tbody>
</table>

The coil with an air core (pink curve) has reasonably constant inductance up to about 2.5 MHz, where it is about 5% above its low frequency value. At higher frequencies the inductance rises rapidly as the frequency approaches 10.5 MHz, the Self-Resonant Frequency (SRF).

A tube of water was added to this coil (described in Appendix A3.3) and the SRF dropped to 6.86 MHz, as shown by the blue curve. The inductance at 2.5 MHz was now 15% above its low frequency value. Water was used for this example because it has a high dielectric constant of 78 and so the effect is easily seen.

The rising inductance is due to the fact that the coil is actually a transmission-line and resonance occurs when the wave reflected from the far end returns in phase with that at the sending end. Adding the water reduces the phase velocity so that resonance now occurs at a lower frequency.

In principle the effect is predictable from transmission-line theory if the $Z_0$ of the line is known along with the phase velocity, but equations for these are not very well developed. Fortunately there is an alternative approach based upon the assumption that the inductance is constant with frequency and that it has self-capacitance across its terminals, and the effect of this capacitance is an apparent change of inductance with frequency.
In this representation the coil has a fixed inductance at all frequencies, and this inductance is equal to its low frequency value (ie low compared to its SRF). The capacitor also has a fixed value, chosen to produce a change in reactance at the terminals which matches that of the real inductor.

Welsby (ref 2) has analysed this equivalent circuit and derives the following equation for the measured inductance (for Q > 3):

$$L_m = \frac{L_o}{1 - (f/f_r)^2}$$  \hspace{1cm}  (2.2.1)

where

- $L_m$ is the inductance measured at frequency $f$
- $L_o$ is the low-frequency inductance
- $f_r$ is the self-resonant frequency (SRF).

This Equation is plotted below for the two values of SRF, with the inductance normalised to its low frequency value:

Correlation between the above equation and the measurements is within 5% up to frequencies of about 75% of the SRF. Since a coil would generally be used at frequencies far below the SRF this correlation is more than acceptable. The exception is antenna loading coils at the lower HF frequencies where operation close to the SRF is common.

The self-resonant frequency $f_r$ in the above equation is given by the following for a coil with one end grounded (ref 1):

$$f_r = \frac{\sqrt{L_o C}}{2\pi}$$
$f_r \approx \left[ 300 \times 0.25 / t_w \right]^{0.8} \times \left[ d_c^2 / (73 \ p) \right]^{0.2} / (\varepsilon'_\text{eff})^{0.5}$ MHz \hspace{1cm} 2.2.2

\[
\text{where } t_w' = t_w \left( 1 + 0.225 \frac{d_c}{t_c} \right) \\
t_w \text{ is the length of wire in the winding} \\
d_c \text{ is the coil diameter to the centre of the wire} \\
p \text{ is the pitch of the winding} \\
\varepsilon'_\text{eff} \text{ is the effective dielectric constant of the former}
\]

2.3. Resistive Loss due to Dielectric

It is shown in Appendix 9 that the dielectric loss can be represented by a resistance in parallel with the coil of value:

$$R_{dp} = \left[ 2\pi f_r^2 L_\text{o} \right] \frac{[\varepsilon' / \varepsilon'']}{[f (1 - 1/\varepsilon'_\text{eff})]} \hspace{1cm} 2.3.1$$

Where $\varepsilon'_\text{eff}$ is the effective dielectric constant

$[\varepsilon' / \varepsilon'']$ is the Q of the dielectric

$L_\text{o}$ is the low-frequency inductance

The Q of the dielectric is usually independent of frequency in the RF range and so $R_p$ reduces with frequency.

Often it is more convenient to express this loss as a series resistance, and the conversion is given by:

$$R_{ds} = \left( R_{dp} X \right) / \left( R_{dp}^2 + X^2 \right) \hspace{1cm} 2.3.2$$

Where $R_{dp}$ is given by Equation 2.3.1

$X = (2\pi f L_m)$

$L_m = L_\text{o} \left[ 1 - (f / f_r)^2 \right]$ 

$L_\text{o}$ is the low-frequency inductance

2.4. Combined Wire Loss and Dielectric Loss: Parallel Resistance

The overall coil loss can be modelled as a parallel resistance $R_p$, comprising the loss due to the dielectric $R_{pd}$, and that due to the conductor $R_{pw}$. The overall parallel resistance is then given by:

$$R_p = \left( R_{dp} R_{wp} \right) / \left( R_{dp} + R_{wp} \right) \hspace{1cm} 2.4.1$$

Where $R_{dp}$ is given by Equation 2.3.1

$R_{wp}$ is given by Equation A10.2

An example is shown below, along with separate curves for $R_{pd}$ and $R_{pw}$. The SRF in this case was 5.35 MHz, and the dielectric Q was 29.
There is an increasing error at high frequencies between the prediction and the measurement, and this is largely due to measurement error since the VNA was not accurate at these very high resistance values (80kΩ). Notice that the parallel resistance due to the dielectric loss is constant with frequency in this case (blue line), because the Q of the water was found to be proportional to frequency (see Appendix 12).

2.5. Combined Wire Loss and Dielectric Loss : Series Resistance

The overall coil loss can also be modelled as a series resistance $R_s$, comprising the series loss due to the dielectric $R_{sd}$, and that due to the conductor $R_{sw}$. The overall resistance is then given by:

$$R_s = (R_{sd} + R_{sw})$$  \hspace{1cm} 2.5.1

where $R_{sd}$ is given by Equation 2.3.2

$R_{sw}$ is given by Equation A10.1

An example is shown below:
The predicted values are shown in red and the measurements in purple (see Appendix 11 for details). The correlation is very close and generally within ± 5% over the 3 octave range of resistance. Also shown is the series resistance due to the wire (brown) and that due to the dielectric (blue). It should be noted that the effect of the dielectric is unusually large in this example (by a factor of up to 100) because the dielectric constant was much higher at 78 compared with a normal former of say less than 2, and the dielectric Q was very low at 45.

3. PUBLISHED INFORMATION ON ELECTRIC FIELD INSIDE A COIL

3.1. Introduction
The above equations require the effective dielectric constant of the winding former \( \varepsilon'_{\text{eff}} \), and this is dependent upon the intensity of the electric fields within the coil. There are three possible electric fields: a radial field which goes across the diameter of the coil, a longitudinal field which follows the axis of the coil, and a circumferential field which is circular, following the wire. The author has been able to find only a few references to field intensity and these are summarized below.

3.2. Cutler
Cutler (ref 3) has carried out some detailed measurements, and the results of these are shown below along with his theoretical predictions:

![Diagram of electric field strengths around a helix. The solid lines are theoretical and the points are experimental. The probe was moved between turns, to the right, and directly opposite a turn of wire at the left.]

Figure 3.2.1 Cutler’s measurements
This shows the longitudinal field to be the largest, decreasing approximately exponentially from a maximum at the centre of the wire to a minimum, non-zero, value at the axis. The radial field is smaller at the wire and then drops to zero at the axis. As for the circumferential field Cutler had difficulty in measuring this and warns that his measurements are not accurate.
In evaluating Cutler’s results it is important to realise that he was not measuring a normal inductance coil, but one intended for accelerating electrons in a Travelling Wave Tube. As such his measurement frequency was very high and the length of each turn corresponded to about $\lambda/4$, whereas in inductance coils the whole length of the wire (ie N turns) would be somewhat less than $\lambda/4$. The voltage across each turn would therefore be smaller by at least $1/N^{th}$ of that in his coil, so if there are 10 turns we could expect the radial field to be $1/10^{th}$ the value he measured, making it negligible compared with the longitudinal field (sometimes called the axial field).

Cutler’s coil had gaps between turns and so he was able to measure the fields both between turns and opposite turns (see Figure 3.2.1). Taking the average of these and normalising to unity gives the following field intensity (blue):

![Figure 3.2.2 Cutler’s Longitudinal Measurement](image)

Also shown is an empirical equation which matches his data (red), and the equation for this is:

$$e = E [(r/r_0)^n + A] \quad \text{v/m}$$

where $n = 3.6$ and $A = 0.38$  \hspace{1cm} 3.2.1

When normalised for $e = 1$ at $r/r_0 =1$ this becomes:

$$e/E = [(r/r_0)^n + A] / (1+A) = [(r/r_0)^{3.6} + 0.38] / 1.38 \quad \text{v/m}$$  \hspace{1cm} 3.2.2

3.3. Lee et al

The assumption that the longitudinal field (or axial field) is the strongest is confirmed by the experiments of Lee et al (ref 4), on an inductance coil and their results are illustrated below:
Payne: The Effect of Dielectric Inside an Inductance Coil

They say ‘The field strength is generally weaker inside the solenoid than outside, because the fields ……are largely cancelled inside. Inside the solenoid, the field directions are mostly axial except near the ends. The radial component decreases rapidly approaching the axis of the solenoid, while the axial component remains relatively constant. On the axis of the solenoid, the field direction is purely axial except near the hot end, as it should be considering the symmetry of the coil’

They also say that no azimuthal component (ie circumferential) is observed either inside or outside within experimental error. One unexpected result is that they show the electric field going to zero towards the top end of the coil, before rising again at the top and beyond, as shown below

‘The experimental result shows the axial field has two peaks, one near the centre of the solenoid and the other above the hot end. The axial component of the electric field becomes zero near the hot end……and the sign of the total axial field changes crossing this position’. They explain this effect by a linear charge distribution along the length of the coil, however it is difficult to understand how this would lead to a field
in the opposite direction. Such a null might be explained by resonance but in their experiment the coil had 15 turns, of diameter 50 mm, and length 72 mm long, and this will have a first SRF of 38.1 MHz (Equation 2.2.2). Their measurement frequency was 3.92 MHz, so much below the SRF.

In summary they say ‘The major field component inside a solenoid is confirmed to be the axial field’.

3.4. Knight

One of the difficulties in measuring the electric field is that most detectors have metallic parts and these affect the electric field being measured. Lee et al (above) used a piezoelectric detector inserted into the coil via a quartz rod which transmitted the data mechanically. Knight (ref 5) avoids interference with the field by using gas discharge tubes to display the field intensity, and his website contains many photographs of these illuminated tubes. His experiments are not able to quantify the field intensity but the following comments by him are useful: ‘This photograph shows that the field is strongest near the helical conductor, and weak along the solenoid axis’ and ‘It appears to confirm that the e-field is tilted, ie, the low-field region at the end of the coil is funnel-shaped’. Significantly in Knight’s photographs there is no evidence of Lee’s null within the coil (Knight’s photos do show nulls but only when the coil is excited at frequencies above its first SRF).

3.5. Choy

Choy (ref 6) gives the following equation for the effective dielectric constant at low frequencies for a former with a fractional volume filling η and relative dielectric constant ε_r:

\[ \varepsilon_{\text{eff}} = 1 + \frac{\eta}{(a+b)} \]

where

\[ a = \frac{1}{(\varepsilon_r-1)} \]
\[ b = \frac{(1-\eta)/2}{(1-\eta)/2} \]

He also gives a related equation for high frequencies nearer the SRF. Neither of these equations includes the wavelength but it is shown by Sichak (below) that the effective dielectric is very dependent upon the wavelength, and this agrees with the author’s experiments.

3.6. Sichak

No closed solution is known for the intensity of the electric fields within a coil. However, Sichak has analysed a similar problem, that of a coaxial transmission-line with a helical inner conductor (ref 7). He considered the effect of a dielectric material inside the helix and a different dielectric between the helix and the outer conductor. In relation to this he says ‘The significant parameter is \((2\pi a N)(2\pi a/\lambda)\), where \(N\) is the number of turns per unit length, \(a\) is the mean radius of the helix and \(\lambda\) the wavelength. When this parameter is considerably less than 1 ……the dielectric inside the helix has only a second order effect, while the dielectric outside the helix has first order effect’. When the parameter above is greater than 1, the effective dielectric constant \(\varepsilon'\) is:

\[ \varepsilon'_{\text{eff}} = \left( \varepsilon_x + \varepsilon_i \right)/2 \]

where \(\varepsilon_x\) is the external dielectric constant, and \(\varepsilon_i\) the internal.

So Sichak defines the two limits which the effective dielectric constant can have, the lower limit being the external \(\varepsilon_x\), and the upper limit given by the above equation. Between these limits the actual value is dependent upon his factor \(F_s = (2\pi a N)(2\pi a/\lambda)\) and this is illustrated below.
In the above curve \( \varepsilon_x \) is assumed to be unity (ie air) and the effective dielectric constant of the core is assumed to be 4, so that Equation 3.6.1 gives the upper limit as \( (1+4)/2 = 2.5 \). The empirical curve has no theoretical basis and is given for illustration only.

Note that Sichak’s factor \( F_s = (2\pi a N)(2\pi a/\lambda) \) can be more conveniently expressed as \( F_s = (\pi d_c)^2/(p \lambda) \) where \( d_c \) is the coil diameter and \( p \) the pitch of the winding.

Unfortunately Sichak does not give an equation for the effective dielectric constant over the transition region from ‘considerably less than 1’, to greater than 1, and this is the region of most interest here. However from his statement we know it changes from \( \varepsilon' = (\varepsilon_x + \varepsilon_i)/2 \) when \( F_s > 1 \), to \( \varepsilon_x \) when \( F_s \) is ‘considerably less than 1’. An empirical equation which fits this criteria is for the effective dielectric constant to be:

\[
\varepsilon'_{\text{eff}} = \frac{(\varepsilon_x + \psi)}{2}
\]

where \( \psi \approx \left[ \varepsilon_{\text{di}} (F_s)^\beta + \alpha \varepsilon_x \right]/\left[ (F_s)^\beta + \alpha \right] \)

\( \varepsilon_x \) is the dielectric constant outside the coil

\( \varepsilon_{\text{di}} \) is the dielectric constant inside the coil (Section 4)

\( \alpha = 4.2 \) (determined from experiments)

\( \beta = 1.1 \) (determined from experiments)

\( F_s = (\pi d_c)^2/(p \lambda_0) \)

\( d_c \) is the mean radius of the coil to the centre of the conductor

\( p \) is the pitch of the winding

\( \lambda_0 \) is the free space wavelength = 300/f for \( f \) in MHz

Generally for an inductance coil there is air outside the helix and so \( \varepsilon_x = 1 \).

The wavelength \( \lambda \) in Sichak’s equation is assumed to be that on the wire. However it is more convenient if \( F_s \) is expressed in terms of the free-space wavelength \( \lambda_0 \) and it is shown in Appendix 7 that \( F_s \) then becomes equal to 0.74 \( [(\pi d_c)^2/(p \lambda_0)]^{1.33} \). However, given that \( \alpha \) and \( \beta \) in Equation 3.6.2 are determined empirically from measurements we can define \( F_s \) in terms of \( \lambda_0 \) as \( F_s = (\pi d_c)^2/(p \lambda_0) \) and chose \( \alpha \) and \( \beta \) accordingly.

The change in wavelength when ferrite is introduced is discussed in Section 5.

### 3.7. Coil Diameter \( d_c \)

There is potentially an uncertainty in the coil diameter as there is for the determination of inductance, especially when the wire diameter is large. However clarification comes from Sichak’s statement that 

\( \lambda_0 \) is the free space wavelength = 300/f for \( f \) in MHz

Generally for an inductance coil there is air outside the helix and so \( \varepsilon_x = 1 \).

The wavelength \( \lambda \) in Sichak’s equation is assumed to be that on the wire. However it is more convenient if \( F_s \) is expressed in terms of the free-space wavelength \( \lambda_0 \) and it is shown in Appendix 7 that \( F_s \) then becomes equal to 0.74 \( [(\pi d_c)^2/(p \lambda_0)]^{1.33} \). However, given that \( \alpha \) and \( \beta \) in Equation 3.6.2 are determined empirically from measurements we can define \( F_s \) in terms of \( \lambda_0 \) as \( F_s = (\pi d_c)^2/(p \lambda_0) \) and chose \( \alpha \) and \( \beta \) accordingly.

The change in wavelength when ferrite is introduced is discussed in Section 5.
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……. dielectric inside the helix has only a second order effect, while the dielectric outside the helix has first order effect’. The dividing line between the inside and the outside of the helix is thus along the centre of the conductor, and this is confirmed by Figure 3.2.1. So for all the equations the coil diameter $d_c$ is defined as that diameter which runs through the centre of the conductor.

Support for this is given by the Alumina experiment described in Appendix 3 where the wire diameter was very large at 17% of the coil diameter, and agreement between the prediction and experiment was 2%.

3.8. Summary of Published Information on Electric Field

In the above references Sichak gives the dependence of the effective dielectric constant on the coil diameter, its winding pitch and the wavelength, leading to Equation 3.6.2. This equation alone would be sufficient if the dielectric former totally filled the winding, but often the former is tubular and so consideration must be given to the change of field intensity across the coil diameter, and this is supplied by Cutler (Equation 3.2.2). These equations are used in the following Section to derive the effective dielectric constant of a former, be this solid, tubular or grooved to locate the wire.

4. DIELECTRIC FORMERS

4.1. Introduction

The dielectric constant of the material making the former is diluted by an inevitable gap between the centre of the conductor and the outer surface of the former (material C in Figure 4.2.1 below). Also the former is often tubular and so the overall dielectric constant is further reduced. For convenience here these reductions are called the ‘dilution’ of the dielectric.

4.2. Dilution of Dielectric Constant

The geometry of a tubular former and its winding is shown below:

![Figure 4.2.1 Tubular Former and wire](image)

In this diagram the tubular former is B, of radii $r_1$ and $r_2$, having an air core A. Clearly if the winding former is solid then $r_1=0$.

The material C consists of a mixture of materials: the conductors, their insulation and air gaps. No electric field can exist in the conductors and so the dielectric is the combination of the air and the insulation. The
latter is likely to be polyamide with a thickness of about 7% of the wire diameter, and a dielectric constant of 2.5. The dielectric constant of the combination will depend upon the ratio of the wire spacing to the wire diameter, but assuming this ratio is similar to the illustration above it can be seen that the field is probably 70% in air. The overall dielectric constant of material C is therefore assumed to be around 1.1.

Note that the longitudinal electric field in the gap between wires is not normally affected by the winding former (be it tubular or solid) and so the dielectric constant of material C is always close to unity. The exception is where the former is grooved to locate the winding and then the dielectric constant of material C is increased.

A theoretical analysis of this configuration is given in Appendix 2 assuming that the electric field is longitudinal. That is, there is no radial component of the field so the field in one layer does not pass across the boundary into the adjacent layer. This seems to be essentially true in the centre of a long coil since the analysis yields results which agree well with measurements, but this cannot be true at the ends and this is covered in Paragraph 4.3.

Equation A2.1 gives the diluted dielectric constant, where \( n \) and \( A \) describe the field intensity across the diameter. If it is assumed that the intensity of the electric field is given by Equation 3.2.2 (ie Cutler’s measurements) and \( \varepsilon_{\text{gap}} = 1.1 \) then the diluted dielectric constant for \( \varepsilon_{\text{air}} = 1 \), becomes:

\[
\varepsilon_{\text{di}} = 1.1 \kappa_g + \varepsilon_{\text{former}} \kappa_l + \kappa_a
\]

where \( \varepsilon_{\text{di}} \) is the diluted dielectric constant

\[
\kappa_g = 1 - 0.5 (r'_2^{5.6} + r'_1^{5.6})
\]

\[
\kappa_l = 0.5 (r'_2^{5.6} + r'_1^{5.6} - r'_1^{5.6} - r'_2^{5.6})
\]

\[
\kappa_a = 0.5 (r'_1^{5.6} + r'_1^{5.6})
\]

The radii \( r' \) are normalised to the coil radius \( r_3 \), and so \( r'_1 = r_1/r_3 \) and \( r'_2 = r_2/r_3 \).

### 4.3. End Effect

Equation 4.2.1 assumes that the coil is very long compared with its diameter, so that the field intensity can be assumed to be constant down the length. In fact the intensity of the electric field reduces towards the ends of the coil, and this further dilutes the effective dielectric constant of the winding former. A simple model of this field variation which agrees with experiment is for it to be constant at unity in the centre section and that it reduces to 0.42 at the ends over a length of 0.22 \( d_c \) from each end (see Appendix 5). Notice that the length over which the field is reduced is assumed to be proportional to the coil diameter \( d_c \), and also that it applies to each end whether this is connected to ground or not, and experiments support these assumptions. Experiment shows that the diluting effect of this field taper is given by:

\[
\text{Longitudinal dilution} = 1 - 0.25 \left[ d_c/l_c \right]
\]

where \( d_c \) is the coil diameter to the centre of the wire

\( l_c \) is the coil length

To use this equation, the diluted dielectric constant Equation 4.2.1 is multiplied by the above factor.

### 4.4. Dielectric Extension

Equation 4.2.1 assumes that the dielectric former has the same length as the coil. In practice the former is usually slightly longer and experiment shows that for a coil earthed at one end, extending the former at this end has no measurable effect. At the other end (the hot end) the effect of the extension is shown below (see Appendix 4 for details):

\[
\]
To use the above curve, the dielectric constant $\varepsilon_{\text{eff}}$ is calculated from Equations 3.6.2 and 4.2.1, and then the calculated value is multiplied by the above factor. An equation for the above is given in Appendix 4, however for short extensions of less than one dielectric diameter, this can be simplified. The total effective dielectric constant $\varepsilon_{\text{total}}$, including the end extension, is then given by:

$$
\varepsilon_{\text{total}} \approx \varepsilon_{\text{eff}} \left[ 1 + 0.045 \left( l_{\text{die}} / d_{\text{die}} \right) \right]
$$

where $l_{\text{die}}$ is the length of the dielectric protrusion at one end $d_{\text{die}}$ is the diameter of the dielectric protrusion $\varepsilon_{\text{eff}}$ is given by Equation 3.6.2

This equation is for a coil grounded at one end, so having one ‘hot’ end only. If the coil is balanced so that both ends are ‘hot’ the equation becomes $\varepsilon_{\text{total}} \approx \varepsilon_{\text{eff}} \left[ 1 + 0.09 \left( l_{\text{die}} / d_{\text{die}} \right) \right].$

Notice that for a normal extension of say 0.5 $d_{\text{die}}$ the correction is only 2% and can be ignored.

4.5. Experimental Support

Experimental support for Equations 3.6.2, 4.2.1 and 4.3.1 is given by the results illustrated below for a range of dielectrics:

a) A coil filled with distilled water dielectric ($\varepsilon_r = 78$)
b) A coil filled with a PTFE dielectric ($\varepsilon_r = 2.3$)
c) A coil with a tube of distilled water
d) A coil with a tube of ceramic (alumina) $\varepsilon_r = 9.68$.

For each experiment the length of the dielectric former was the same as that of the coil, so the correction provided by Equation 4.4.1 was not required. The theoretical curve given in the following graphs is given for $\alpha = 4.2$ and $\beta = 1.1$ (Equation 3.6.2), values optimized to give the best agreement with all the experiments (see Appendix 3 for details of these experiments).

Each graph shows three or four experimental points, but these are not for three different coils but for the same coil/dielectric combination but at the first three/four resonant frequencies.
Coil Filled with distilled water:

![Graph of Effective Permittivity with Water Dielectric](image)

*Figure 4.5.1 Coil Filled with water of same length as Coil*

Notice that both scales are logarithmic. The diluted dielectric constant was 56 (Equations 4.2.1 and 4.3.1).

Coil Filled with solid PTFE:

![Graph of Effective Permittivity with PTFE Dielectric](image)

*Figure 4.5.2 Coil Filled with PTFE of same length as Coil*

Notice that the x axis is linear. The diluted dielectric constant was 2.17 (Equations 4.2.1 and 4.3.1).
Coil with tube of distilled water:

![Effective Permittivity 0.73d Hole in Water Dielectric](image)

*Figure 4.5.3 Tubular Former of Water*

Notice that both scales are logarithmic. The dielectric constant was 36 (Equations 4.2.1 and 4.3.1).

Coil with tube of Alumina:

![Effective Permittivity with Ceramic tube Dielectric](image)

*Figure 4.5.4 Tubular Alumina Former*

Notice that the x axis is linear. The diluted dielectric constant was 3.7 (Equations 4.2.1 and 4.3.1).

There is a trend in the graphs above for the agreement between experiment and theory to be better at the first resonance than at the higher resonances but the reason for this not known.

The values chosen for n and A in the above curves were 3.6 and 0.38 respectively (describing the electric field intensity), and these gave good agreement with experiment and have a sound basis from Cutler’s measurements (albeit with different values of α and β). However the values are not critical, and a wide range
of alternatives give equally good correlation with experiment. The reason for this is that, even if the field intensity was constant across the coil, the displacement current reduces as the enclosed area, so the current in the centre of the coil is very much reduced over that near the winding. For instance at half the radius ($r/r_o = 0.5$), the area enclosed is only $1/4$ of the total and so dielectric material at larger radii will have 3 times the effect of dielectric in the centre. This area factor greatly diminishes the effect of changing the rate at which the field decreases.

5. FERRITE CORES

Ferrites have a dielectric constant with a value of around $\varepsilon_r \approx 10$. In applying the previous equations account needs to be made for the effect of the permeability, and this changes the wavelength on the wire. Sichak’s factor then becomes:

$$F_S = \frac{(\pi d_c)^2}{(p \lambda_o) c/V_w}$$

where $V_w/c$ is the wire velocity as a proportion of $c$

When the ferrite is introduced the ratio $c/V_w$ is equal to $\sqrt{\mu_{coil}}$, where $\mu_{coil}$ is the effective permeability of the ferrite, and is equal to the ratio of the inductance with and without the ferrite. For a toroid $\mu_{coil} = \mu_r$, the relative permeability of the ferrite. For an open core such as a ferrite antenna or the experiment here, the permeability is diluted because the flux has to flow partially in air. This is discussed in the author’s reference 8. So Equation 5.1 becomes:

$$F_S = \frac{(\pi d_c)^2}{(p \lambda_o) (\mu_{coil})^{0.5}}$$

where $\mu_{coil}$ is the ratio of low frequency inductance with and without the ferrite

d_c is the coil diameter to the centre of the wire
p is the winding pitch
$\lambda_o$ is the free-space wavelength = $300/f$ where $f$ is in MHz.

Appendix 8 describes an experiment to test this equation with $\mu_{coil} = 4$ with the following results:

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Figure 5.1 Tubular Alumina Former}
\end{figure}

The pink curve is that of Equation 3.6.2, with $\alpha = 4.2$ and $\beta = 1.1$ (ie the same as all other experiments). $F_S$ for the measurement point was calculated from Equation 5.2. Agreement is better than 2%.
6. SUMMARY OF EQUATIONS
The Q of a coil is given by (Equations 2.2.1 and 2.5.1):

\[ Q = \frac{\omega L_0}{[1 - (f/f_r)^2]} \frac{1}{(R_{ds} + R_{ws})} \]

**Equation 6.1**

where

\[ L_0 \] is the low frequency inductance

\[ f_r \approx \left[ \frac{300 \times 0.25}{l_w} \right]^{0.8} \left[ \frac{d_c^2}{(73 \, \text{p})} \right]^{0.2} (\varepsilon'_{\text{eff}})^{0.5} \text{ MHz} \]

**Equation 2.2.2**

where

- \( l_w \) is the length of wire in the winding
- \( d_c \) is the coil diameter to the centre of the wire
- \( p \) is the pitch of the winding
- \( \varepsilon'_{\text{eff}} \) is the effective dielectric constant of the former

\[ \varepsilon'_{\text{eff}} = \frac{\varepsilon_x + \psi}{2} \]

**Equation 3.6.2**

where

- \( \psi \approx [\varepsilon_{di} (F_S)^{\beta} + \alpha \, \varepsilon_x] / (F_S)^{\alpha} \)
- \( \varepsilon_{di} \) is the dielectric constant outside the coil (normally =1)
- \( \varepsilon_{di} \) is the dielectric constant inside the coil (see below)
- \( \alpha = 4.2 \) (determined from experiments)
- \( \beta = 1.1 \) (determined from experiments)
- \( F_S = \frac{(\pi d_c)^2}{p \, \lambda_0} \)
- \( d_c \) is the mean radius of the coil to the centre of the conductor
- \( p \) is the pitch of the winding
- \( \lambda_0 \) is the free space wavelength = 300/f for f in MHz

\[ \varepsilon_{di} = (k_g + \varepsilon_{\text{former}} \, k_f + \, k_a) \left[ 1 - 0.25 \left( \frac{d_c}{l_c} \right) \right] \]

**Equations 4.2.1 & 4.3.1**

where

- \( k_g = 1 - 0.5 \left( r_{2,5,6}^2 + r_{2,5,6}^2 \right) \)
- \( k_f = 0.5 \left( r_{2,5,6}^2 + r_{2,5,6}^2 - r_{1,5,6}^2 - r_{1,5,6}^2 \right) \)
- \( k_a = 0.5 \left( r_{1,5,6}^2 + r_{1,5,6}^2 \right) \)
- \( \varepsilon_{\text{former}} \) is the dielectric constant of the former
- \( d_c \) is the coil diameter to the centre of the wire
- \( l_c \) is the coil length

The radii \( r' \) are normalised to the coil radius \( r_3 \), and so \( r_1' = r_1 / r_3 \) and \( r_2' = r_2 / r_3 \).

\[ R_{ds} = \frac{(R_{dp} \, X^2)}{(R_{dp}^2 + X^2)} \]

**Equation 2.3.2**

where

- \( X = (2\pi f L_{\text{in}}) \)
- \( L_{\text{in}} = L_0 / [1 - (f / f_r)^2] \)
- \( L_0 \) is the low-frequency inductance
Payne: The Effect of Dielectric Inside an Inductance Coil

\[ R_{dp} = \left[ 2\pi f_1^2 L_0 \right] \left[ \varepsilon' / \varepsilon'' \right] / \left[ f (1 - \varepsilon'_{\text{eff}}) \right] \]  \hspace{1cm} (Equation 2.3.1)

where \( \left[ \varepsilon' / \varepsilon'' \right] \) is the Q of the dielectric

\[ R_{ws} = K \sqrt{f / \left[ 1 - (f / f_r)^2 \right]} \]  \hspace{1cm} (Equation A10.1)

where \( K = R_{dc} \phi / 0.25 \) \( d_w / 0.067 \)
\( R_{dc} = 4 \rho \ell_w / (\pi d_w^2) \)
\( d_w \) is dia of the wire in metres
\( \rho \) = resistivity (1.77 \( 10^{-8} \) for copper)
\( \ell_w \) is the length of the conductor (straight length) in metres
\( \phi \) is the proximity effect (see ref 1 Appendix 1, and note below).

NB if the ratio of the wire diameter to the pitch is 0.5 (which often gives the highest Q) then \( \phi \approx 1.78 \) for coils with a length equal to twice the diameter or longer.

7. PRACTICAL APPLICATIONS

Antenna loading coils are often operated at frequencies close to the SRF, and then the effect of the dielectric is greatest. The following example of a loading coil has been extracted from an ARRL handbook for resonating an 8ft whip at 3.8 MHz with the coil at the centre of the whip:

Coil length : 254 mm
Coil dia : 65 mm
Number of turns : 100
Wire diameter : 1.3 mm (16 AWG) (giving \( d_w / \rho = 0.5 \))
Low frequency inductance \( L_0 = 140 \mu\text{H} \) (calculated from Wheeler’s equation)
SRF : 5.8 MHz (calculated from Equation 2.2.2)

If there is no dielectric former, there will be only conductor loss and the coil Q is then calculated to be (from Equation 6.1 with \( R_{ds} = 0 \)):

![Predicted Q v Frequency](image)

*Figure 7.1.1 Coil Q with only conductor loss*
Notice that the Q peaks at around half the SRF, and at the operating frequency of 3.8 MHz the Q has dropped to about 86% of this peak value. If a lossless tubular former is introduced having a dielectric constant of 3 and a wall thickness 10% of the coil diameter, the Q becomes:

![Predicted Q v Frequency](image)

*Figure 7.1.2  Coil Q with lossless dielectric former*

Notice that although the former is lossless the Q has dropped significantly at 3.8 MHz, because of the decrease in the SRF.

If this former has loss with a dielectric Q of 100, the overall coil Q becomes (red curve):

![Predicted Q v Frequency](image)

*Figure 7.1.3  Coil Q with lossy former*

Notice that when the dielectric former is introduced the greatest reduction in Q is due to the reduction in the SRF, rather than the loss in the dielectric, even though the dielectric was assumed to be of relatively low Q. So formers with a high dielectric constant should be avoided even if they have high Q (e.g. Ceramic), and indeed a dielectric with a poor loss tangent can perform better if its dielectric constant is very low.
8. TEST EQUIPMENT AND CALIBRATION

All measurements were made with an Array Solutions UHF Vector Network Analyser. Calibration of this analyser required an open circuit, a short circuit and known resistive load, and these are shown below.

![Figure 8.1 Calibration loads](image)

To ensure that the resistive load had minimal stray reactance a thick-film resistor was used, and this had the added advantage that it could be located in the same plane as the short circuit. Its value was 47 Ω ± 1%. SMA connectors were used because they are small and therefore have a small stray capacitance, and so any error in calibrating this out would also be small.
APPENDIX 1: DILUTED DIELECTRIC CONSTANT OF A SOLID FORMER

Consider a coil with dielectric cylinder inside having a radius \( r \), thickness \( \delta \), and a length equal to that of the coil. Given that the main electric field is longitudinal this will be along the length of the cylinder, so the displacement current will also be in this direction. For this cylinder the displacement current will be proportional to the cross-sectional area \( 2\pi r \delta \), and to the electric intensity \( e \):

\[
i = e \varepsilon_n \varepsilon_r \frac{2\pi r}{\varepsilon_o} \text{ dr}
\]

where \( e \) is the electric intensity in \( \text{v/m} \)

From Cutler the intensity will have an intensity similar to that below:

\[
e = E (r^n + A) \text{ v/m}
\]

where \( E, n \) and \( A \) are constants

Combining the above two equations gives:

\[
i_{de} = E \varepsilon_n \varepsilon_r \frac{2\pi (r^n + A)}{\varepsilon_o} \text{ dr}
\]

\[
= \varepsilon_n \varepsilon_r \frac{2\pi E (r^{n+1} + A r)}{\varepsilon_o} \text{ dr}
\]

The total longitudinal current between a radius of \( r_1 \) and \( r_2 \) will be:

\[
I_{de} = \frac{2\pi E \varepsilon_n \varepsilon_r}{\varepsilon_o} \int_{r_1}^{r_2} (r^{n+1} + A r) \text{ dr}
\]

\[
= 2\pi E \varepsilon_n \varepsilon_r \left\{ r_2^{(n+2)/(n+2)} + A r_2^{n+2}/2 \right\} - \left\{ r_1^{(n+2)/(n+2)} + A r_1^{n+2}/2 \right\}
\]

To find the dielectric constant this displacement current can be compared to that with no dielectric \( I_{d1} \), and this is given by the above equation when \( \varepsilon_r =1 \), \( r_2 = r_o \) (the radius of the coil) and \( r_1 =0 \):
\[ I_{d1} = 2\pi E \varepsilon_0 \left[ r_0^{(n+2)} / (n+2) + A r_0^2 / 2 \right] \quad A1.5 \]

So the effective dielectric is the ratio of the currents with and without the dielectric:

\[ e_{\text{eff}} = \frac{r_2^{(n+2)} / (n+2) + A r_2^2 / 2}{r_1^{(n+2)} / (n+2) + A r_1^2 / 2} \quad A1.6 \]

It is convenient to normalise all radii to that of the coil \( r_0 \), so Equation A1.6 becomes:

\[ e_{\text{eff}} = \frac{r_2^{(n+2)} / (n+2) + A r_2^2 / 2}{r_1^{(n+2)} / (n+2) + A r_1^2 / 2} \quad A1.7 \]

where

\[ r_2' = r_2 / r_0 \]
\[ r_1' = r_1 / r_0 \]

APPENDIX 2 : EFFECTIVE DIELECTRIC CONSTANT OF A TUBULAR FORMER

With a tubular former there are three dielectrics rather than the two above, as shown in the diagram below (see also Section 4):

\[ \epsilon_{rt} = \epsilon_{gap} k_g + \epsilon_{form} k_f + \epsilon_{air} k_a \quad A2.1 \]

where

\[ k_g = \frac{r_3^{(n+2)} / (n+2) + A r_3^2 / 2}{r_2^{(n+2)} / (n+2) + A r_2^2 / 2} \]
\[ k_f = \frac{r_2^{(n+2)} / (n+2) + A r_2^2 / 2}{r_1^{(n+2)} / (n+2) + A r_1^2 / 2} \]
\[ k_a = \frac{r_1^{(n+2)} / (n+2) + A r_1^2 / 2}{1 / (n+2) + A / 2} \]

Figure A2.1 Dielectrics with Tubular Former

Assuming the electric field is longitudinal, and therefore that the displacement current is longitudinal and does not go from one dielectric to another, then the combined displacement current is the sum through each one and so the combined dielectric constant is (from Equation A1.7):

\[ \epsilon_{rt} = \epsilon_{gap} k_g + \epsilon_{form} k_f + \epsilon_{air} k_a \]
APPENDIX 3: MEASUREMENTS OF EFFECTIVE DIELECTRIC CONSTANT IN COILS

A3.1. Introduction
It is shown in the main text that the effective dielectric constant of a dielectric can be described by Equation 3.6.2, using the diluted dielectric constant determined by Equation 4.2.1. However, there are then four unknown parameters, α and β in the first of these equations and n and A in the second equation. It was intended to determine these by experiment but the parameters are slightly interactive and so as a starting point it was assumed that the electric field was as measured by Cutler (Section 2.2), and this gave n=3.6 and A=0.38. With these values the best agreement with all the following experiments was with α = 4.2 and β = 1.1. So the modeled dielectric constant in the following experiments uses:

\[
\begin{align*}
n &= 3.6 \text{ and } A = 0.38 \text{ in Equation 3.2.1} \\
\text{with } \alpha &= 4.2 \text{ and } \beta = 1.1 \text{ in Equation 3.6.2.}
\end{align*}
\]

It is shown that these values give good agreement with experiment, but equally it is found that these values were not critical and a wide range of alternatives give equally good correlation with experiment.

The experiments measured the SRF of coils with both an air core and a dielectric core, both solid and tubular. The change in SRF was assumed to be due to the dielectric reducing the phase velocity by \(\sqrt{\varepsilon'}\) (Equation 2.2.1), so that the measured effective dielectric constant was given by:

\[
\text{Measured } \varepsilon' = \left[ \frac{\text{SRF air}}{\text{SRF dielectric}} \right]^{0.5}
\]

In the initial experiments only the first SRF was measured but it was then realised that the higher SRF’s would also give useful data since these higher frequencies would correspond to larger values of the Sichak factor \(F_s\).

All coils were operated with one end grounded, and this excited the \(n\lambda/4\) modes where \(n\) is an integer, 1, 2, 3, 4…. The measured SRF’s are not exact integers as this would imply because of end effect, and also because the effective dielectric constant increases with frequency, as predicted by Figure 3.6.1.

The length of the dielectric was the same length as the coil in each case. In practice the former is likely to be slightly longer and the effect of this is discussed in Section 4.4.

A ground plane soldered to the lower end of the coil was needed for reproducible measurements except for the large coil figure A3.2.1.

Measurements were made with an Array Solutions UHF VNA, coupled into the coil by either a tap on the coil, or alternatively a coupling loop. The SRF’s were defined as those frequencies where the phase of the input impedance went through zero.

A3.2. Total fill of water

Water has a very high dielectric constant of 78, and so the reduction of the SRF will be large, thereby reducing some of the experimental error. Distilled water was used because it has a much lower loss than tap water, but nevertheless the Q at resonance was only around 15 at 12 MHz, and this tended to smear one resonance into another (see below) (Note: the distilled water used here was from a de-humidifier, and it was found that the loss of this water doubled if it was left for several weeks in an open beaker—see Appendix 12).

A coil was wound with 59 turns of 1 mm dia wire, onto a thin plastic bottle of outer diameter 49.2 mm, giving a mean wire diameter of 50.2 mm. The length of the winding was 125 mm and inside diameter of the tube was 47.2 mm. To minimize loading of the coil by the VNA and connection leads, connection to the VNA was at a tap on the coil 16 turns from the earthed bottom end. The top end had no lead and no
connection. The bottle was filled with water to a height of 125 mm, ie the top of the winding. The water extended by 10 mm beyond the bottom (earthed) end of the coil, but it is shown in Appendix A4 that this will have had no significant effect.

![Test Coil ready for filling with Water](image)

*Figure A3.2.1 Test Coil ready for filling with Water*

NB the size of the coil can be judged by comparison with the SMA connector which has on outer metal length of 10mm.

Measurements gave the following (frequencies are in MHz):

<table>
<thead>
<tr>
<th></th>
<th>$\lambda/4$</th>
<th>$\lambda/2$</th>
<th>$3\lambda/4$</th>
<th>$4\lambda/4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRF air</td>
<td>12.38</td>
<td>28.7</td>
<td>41.7</td>
<td>56.3</td>
</tr>
<tr>
<td>SRF water</td>
<td>6.37</td>
<td>9.78</td>
<td>14.03</td>
<td>19.93</td>
</tr>
<tr>
<td>Calculated $\varepsilon'$</td>
<td>3.78</td>
<td>8.63</td>
<td>8.83</td>
<td>7.99</td>
</tr>
<tr>
<td>Sichak $F_s$</td>
<td>0.484</td>
<td>1.124</td>
<td>1.632</td>
<td>2.204</td>
</tr>
</tbody>
</table>
The diluted dielectric constant of the water, $\varepsilon_{dil}$ (Equations 4.2.1 and 4.3.1) was calculated to be 62.5 assuming the effective dielectric constant of the gap between water and the centre of the conductor was 1.1. The measured impedance with water is shown below, for the first four resonances. It is noticeable that the Q at resonance is very low, and this is largely due the loss in the distilled water because the coil without water showed a Q of 64.

A tube of water was made by inserting a 36.4 mm plastic tube into the coil shown in Figure A 3.2.1 and filling around it with distilled water. The inside diameter of the tube of water was 0.73 that of the coil. Measurements gave the following:

**Figure A3.2.2 Measurements of water dielectric cf empirical model.**

**Figure A3.2.3 Measured Impedance with water dielectric**

A3.3.  

36.4 mm Tube of water

A tube of water was made by inserting a 36.4 mm plastic tube into the coil shown in Figure A 3.2.1 and filling around it with distilled water. The inside diameter of the tube of water was 0.73 that of the coil. Measurements gave the following:
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<table>
<thead>
<tr>
<th></th>
<th>λ/4</th>
<th>λ/2</th>
<th>3λ/4</th>
<th>4λ/4</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRF air</td>
<td>12.21</td>
<td>28.2</td>
<td>40.6</td>
<td>55.2</td>
</tr>
<tr>
<td>SRF water</td>
<td>7.82</td>
<td>12.13</td>
<td>16.15</td>
<td>22.09</td>
</tr>
<tr>
<td>ε'</td>
<td>2.44</td>
<td>5.41</td>
<td>6.32</td>
<td>6.24</td>
</tr>
<tr>
<td>Sichak F_s</td>
<td>0.48</td>
<td>1.10</td>
<td>1.59</td>
<td>2.16</td>
</tr>
</tbody>
</table>

Effective Permittivity 0.73d Hole in Water Dielectric

Figure A3.3.1 Measured Impedance with water tube dielectric

The diluted dielectric constant of the water, ε_{dill} was calculated to be 40.3 assuming the effective dielectric constant of the gap between water and the centre of the conductor was 1.1.

A3.4. Solid PTFE

A coil was close wound onto a solid PTFE bar of 14 mm diameter using copper wire of diameter 0.44 mm over its enamel insulation, over a length of 29.3 mm, thus giving approximately 67 turns:

Figure A3.4.1 Coil with PTFE Dielectric (ground-plane not shown)
NB the size of the coil can be judged by comparison with the SMA connector which has an outer metal length of 10 mm.

It was found that a well-defined ground plane was necessary (not shown above) to give repeatable results and this was formed from a thin sheet of copper 150 mm x 110 mm. One end of the coil was connected to this ground plane and also to the earth of the test jig.

The SRF of this coil was to be measured with and without the PTFE and so it was important that the PTFE could be easily removed, while minimising any air gap between it and the winding. This was achieved by unwinding the coil by a fraction of a turn while it was on the rod and then securing it with cellotape (see picture). The wire was cut-off close to the former at the top end and the bottom was connected to ground terminal of the test connector via a short lead of about 5 mm length. Coupling to the coil was via a 5 turn coupling loop at the earthed end of the coil.

Measurements gave the following:

<table>
<thead>
<tr>
<th></th>
<th>(\lambda/4)</th>
<th>(\lambda/2)</th>
<th>(3\lambda/4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SRF air</td>
<td>34.07</td>
<td>73.89</td>
<td>103.68</td>
</tr>
<tr>
<td>SRF water</td>
<td>33.00</td>
<td>68.66</td>
<td>95.83</td>
</tr>
<tr>
<td>(\varepsilon')</td>
<td>1.07</td>
<td>1.16</td>
<td>1.17</td>
</tr>
<tr>
<td>Sichak (F_s)</td>
<td>0.61</td>
<td>1.33</td>
<td>1.87</td>
</tr>
</tbody>
</table>

**Figure A3.4.2 Measured Impedance with solid PTFE dielectric**

The dielectric constant of pure PTFE is 2.1 but often a small proportion of glass is introduced to improve its mechanical properties. This increases the dielectric constant slightly and measurements of the author’s material gave 2.3 (see Appendix 12). The diluted value was calculated as 2.18

**A3.5. Tubular Alumina**

A coil of 30 turns was wound onto an Alumina tube over a length of 30 mm so the pitch was 1 mm. The outer diameter of the tube was 6.25 mm and the inner diameter 4.85 mm. The wire was 20 swg enamelled copper wire having a quoted copper diameter of 0.914 mm and a measured diameter over insulation of 0.98 mm. The thickness of the insulation was therefore \((0.98 - 0.914)/2 = 0.033\) mm.
One potential problem with ceramic materials is that they can be porous and absorb moisture. To test this the ceramic tube was heated to soldering iron temperature for one hour, cooled to room temperature and the
SRF re-measured. The SRF had changed less than 0.1 MHz in 180 MHz, and this is within the experimental repeatability.

APPENDIX 4: DIELECTRIC EXTENSION

A4.1. CERAMIC TUBE

To test the end effect of the coil shown in Figure 3.5.1 the SRF was measured for various protrusions of the ceramic tube from one end of the coil. The tube was very much longer than the coil so that it always protruded a great distance beyond the other end, so that in effect the amount of protrusion at this end was constant. This was repeated for each end, the earthed end and the free end. For completeness the effect on the SRF of the end of the dielectric tube being within the coil was measured. Measurements gave the following:

![Permittivity v Protrusion Ceramic Tube](image)

Figure A4.1.1 Extension of Ceramic Tube Dielectric

In the above the protrusion has been normalized to the coil diameter. The effective dielectric constant has been normalized to the value it has when the former is the same length as the coil. So the values on the y-axis are the multiplier for protrusion.

The measurements show that extending the dielectric beyond the cold end has no effect on the SRF (the pink curve) and so there can be no significant electric field beyond the coil at this end. In contrast there is a larger electric field beyond the free end, with the effective dielectric constant increasing rapidly for short protrusions (brown curve).

Also shown in yellow is a semi-empirical equation described in paragraph 4.4.

A4.2. Solid PTFE

A similar experiment was carried-out with the coil shown in Figure 3.4.1, again with a dielectric much longer than the coil.

Measurements gave the following:
A4.3. Distilled Water

A similar experiment was carried-out with the coil shown in Figure A3.2.1. Distilled water was poured into the bottle to various levels including beyond the end of the coil, with the following results:

Notice that the normalised effective dielectric constant is very similar to the measurements with the ceramic tube and the PTFE, as indicated by the empirical equation plotted for all three. However at large protrusions the water is having a larger effect and this is probably due to its lower electric reluctance (i.e., higher dielectric constant).

A4.4. Equation for Extension

The effect of extending the dielectric beyond the end of the coil is likely to be similar to that of extending magnetic material beyond the end of the coil. This has been analysed by the author (ref 8), assuming that magnetic material inside the coil affects only the internal field, and material outside the coil affects only the external field. The sudden change in slope as the dielectric protrudes indicates that this may also be true for the electric fields.
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In the case of the magnetic field it was shown that as the ferrite is extended beyond the end of the coil the effective permeability increased logarithmically. However with dielectric the extension will be relatively small and so a linear approximation can be assumed. The total effective dielectric constant $\varepsilon_{\text{total}}$, including the extension, can then be expressed by the following empirical equation:

$$
\varepsilon_{\text{total}} \approx \varepsilon'_{\text{eff}} \left[ 1 + k_1 \left( l'_\text{die} / d_\text{die} \right) \right]
$$

where $l'_\text{die}$ is the length of the dielectric protrusion at one end and $d_\text{die}$ is the diameter of the dielectric material

Where $\varepsilon'_{\text{eff}}$ is the effective dielectric constant when the dielectric is the same length as the coil i.e. $l'_\text{die} = 0$, (and given by Equation 4.2.1) and the extension is given in terms of an increase in this effective dielectric constant. Best agreement with the experiments above was with $k_1 = 0.055$.

Normalising this equation with respect to $\varepsilon'_1$ gives:

$$
\varepsilon_{\text{ext}} / \varepsilon'_{\text{eff}} \approx \left[ 1 + 0.055 \left( l'_\text{die} / d_\text{die} \right) \right]
$$

This equation agrees well with the water experiment, but not those with ceramic and PTFE. The reason is that the water has a much higher effective dielectric constant of 78 compared with 3.75 for the ceramic (diluted from 9.6 because it is a tube) and 2.3 for the solid PTFE. This is a similar to the effect of low permeability with magnetic materials, and the reduction in this case was found to be given by the following semi-empirical equation:

$$
\Phi / \Phi_{\text{max}} \approx 1 / \left[ 1 + \left( l'_f / d_f \right)^{1.4} / (k_2 \mu_\text{e}) \right]
$$

where $l'_f$ is the protruding length ($l_f - l_\text{coil}$) $\Phi$ is the electric flux

So in terms of protrusion at one end only this becomes (changing the symbols for dielectric rather than ferrite):

$$
\Phi / \Phi_{\text{max}} \approx 1 / \left[ 1 + \left( 2 \left( l'_\text{die} / d_\text{die} \right) \right)^{1.4} / (k_2 \varepsilon_\text{e}) \right]
$$

where $l'_\text{die}$ is the length of the dielectric protrusion at one end $d_\text{die}$ is the diameter of the dielectric material $\varepsilon_\text{e}$ is the permeability of the extension (diluted if a tube)

Combining Equations A4.4.1 and A4.4.4 gives:

$$
\varepsilon_{\text{total}} \approx \varepsilon'_{\text{eff}} \left[ 1 + 0.055 \left( l'_\text{die} / d_\text{die} \right) \right] / \left[ 1 + \left( 2 \left( l'_\text{die} / d_\text{die} \right) \right)^{1.4} / (k_2 \varepsilon_\text{e}) \right]
$$

The best match with the above experiments was with $k_2$ equal to 1.55, and this equation is plotted in yellow in each of the curves above, and shows good agreement with the ceramic and water experiments.

Agreement with the PTFE experiment is less good but the experimental error is higher here because the coil winding was not mechanically stable and could have changed during insertion of the dielectric.

The above equation can be simplified for a practical situation where the protrusion will be not more than 0.5 $d_\text{die}$ and also the effective material dielectric constant will be close to unity, because a low dielectric constant material will be used and as a tube. The diluted dielectric constant $\varepsilon_\text{e}$ can be assumed to be 1.5. Equation A4.4.5 can then be simplified to:

$$
\varepsilon_{\text{total}} \approx \varepsilon'_{\text{eff}} \left[ 1 + 0.045 \left( l'_\text{die} / d_\text{die} \right) \right]
$$

where $l'_\text{die}$ is the length of the dielectric protrusion at one end $d_\text{die}$ is the diameter of the dielectric material $\varepsilon'_{\text{eff}}$ is given by Equation 3.6.2
APPENDIX 5: MEASUREMENTS OF LONGITUDINAL FIELD TAPER

The longitudinal taper of the electric field was measured with coils of five different ratios of length to diameter of 0.38, 0.5, 1.16, 1.99, and 2.79. Each coil had 18 turns of 1 mm dia wire wound onto a thin plastic bottle of outer diameter 49.2 mm, giving a mean wire diameter of 50.2 mm (i.e. the plastic former shown in Figure A3.2.1).

The lower end of the winding was connected to earth and the VNA was connected to a tap on the coil. The top of the winding had no lead and no connection. At each length the SRF was measured, water was poured into the bottle, and the SRF re-measured with the following results:

**Figure A5.1** The measured dependence of \(\varepsilon'_{\text{eff}}\) on coil length

The empirical curve in pink is arbitrary, and is included merely to illustrate the trend. The two shorter coils clearly have a much lower effective dielectric constant than the trend of the longer coils, and this is due to the field reducing towards the ends. This can be seen from an expansion of Figure A4.3, shown below:

**Figure A5.2** Change of Slope close to end of Coil
It can be seen that close to the end of the coil (ie \( l'_{\text{die}}/d_{\text{die}} = 0 \)) the slope reduces. Since the curve amounts to the integral of the flux \( x \) length, this implies a reduction of flux density towards the end, to around 40% of the central flux over a distance of around 0.2 \( d_{\text{die}} \). Clearly a very short coil with a length say of only 0.4 \( d_{\text{die}} \) will have only 40% of the flux of a longer coil, with a commensurate reduction in the effective dielectric.

To allow for this effect there are three possibilities: a) the Sichak factor \( F_c = (2\pi a N)/(2\pi a/\lambda) \) could be modified, b) Equation 3.6.2 could be modified, or c) the diluted dielectric constant of the dielectric could be modified. The latter seems the most appropriate in that the diluted value is already a function of the radial field intensity, and this would make it now also dependent on the longitudinal intensity. It is shown in Appendix 6 that the diluted dielectric constant is now multiplied by the following:

\[
\text{Longitudinal dilution} = 1 - x \left[ d_c / L_c \right] \left[ 1 - \frac{\varphi_{\text{min}}}{\varphi_{\text{max}}} \right] \tag{A5.1}
\]

The factor \( x \) is the relative distance from end at which the electric flux intensity reduces, and \( \varphi_{\text{min}} / \varphi_{\text{max}} \) is the reduced flux intensity at the end compared with that in the middle of the coil. We have seen that these are approximately equal to \( x = 0.4 \) and \( \varphi_{\text{min}} / \varphi_{\text{max}} = 0.4 \), but the experiment above can give these values more accurately, and it is shown that these are 0.42 and 0.43 respectively. Using these values and Equation A6.4 for each of the lengths gives the dilution as:

<table>
<thead>
<tr>
<th>Coil Length/ Diameter</th>
<th>Longitudinal Dilution</th>
<th>Overall diluted Dielectric Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.38</td>
<td>0.37</td>
<td>23.6</td>
</tr>
<tr>
<td>0.5</td>
<td>0.52</td>
<td>33.4</td>
</tr>
<tr>
<td>1.16</td>
<td>0.8</td>
<td>51</td>
</tr>
<tr>
<td>1.99</td>
<td>0.88</td>
<td>56.5</td>
</tr>
<tr>
<td>2.79</td>
<td>0.91</td>
<td>58.7</td>
</tr>
</tbody>
</table>

(NB in the above the plastic former dilutes the water from \( \varepsilon_r = 78 \) to \( \varepsilon_r = 64.2 \))

Using the new diluted dielectric constant for each length gives:

\[
\text{Effective Permittivity with Water Dielectric}
\]

\[
\text{Figure A5.3 Results with Longitudinal Dilution}
\]

NB A true representation of the results would have a different light-blue line (and thus pink curve) for each measured point, and equal in value to the overall diluted dielectric constant in the table above. However this would be confusing and would not allow direct comparison with Figure A5.1. So in the above figure, all measured values have been increased by the longitudinal dilution as calculated from Equation A5.1, so that the blue points represent the values that would obtain if there was no longitudinal dilution. The pink curve is Equation 3.6.2 with \( \alpha = 4.2 \) and \( n = 1.1 \).
Balanced Coil
The above experiments used coils with one end grounded, and the results indicate that the flux tapering was at both ends (in contrast to the effect of extending the dielectric : Appendix 4). To confirm this an 18 turn coil was wound over a length of 22 mm (length/dia = 0.44) and used in a balanced configuration, whereby the centre was grounded and both ends left free. Water was used as the dielectric. The VNA was connected to a tap on the coil 4 turns from the centre. The coil was also measured with one end grounded, with results two shown below:

![Effective Permittivity with Water Dielectric](image)

The same longitudinal correction was applied to both configurations (Equation and it can be seen to apply to both. So the flux tapering along the length is the same whether one end is grounded or not. The pink curve is Equation 3.6.2 with $\alpha = 4.2$ and $n=1.1$ (ie the same as Figure A5.3).

### APPENDIX 6: LONGITUDINAL FIELD TAPER

In the following analysis it is assumed that the longitudinal flux intensity is $\varphi_{\text{max}}$ down the whole length of the coil except close to each end where it decreases to $\varphi_{\text{min}}$. It is further assumed that the length of this flux reduction is related to the diameter of the coil, so that its length is $x \, d_c$. So assuming the flux intensity is unity over the length $(l_c - x \, d_c)$ and $\varphi_{\text{min}} / \varphi_{\text{max}}$ over a length $x \, d_c$ (ie the length at each end of the coil is 0.5 $x/ d_c$), then the combined flux $\varphi_t$ is equal to:

$$\varphi_t = 1 \left( (l_c - x \, d_c) + (\varphi_{\text{min}} / \varphi_{\text{max}}) (x \, d_c) \right)$$  \hspace{1cm} A6.1

Normalising to the coil length $l_c$ gives :

$$\Phi'_{\text{t}} = [1 - x (d_c / l_c)] + (\varphi_{\text{min}} / \varphi_{\text{max}}) (x (d_c / l_c))$$  \hspace{1cm} A6.2

Where $\Phi'_{\text{t}}$ is the longitudinal dilution of flux. Collecting terms gives :

$$\text{Longitudinal dilution} = 1 - x \left[ d_c / l_c \right] \left[ 1 - \varphi_{\text{min}} / \varphi_{\text{max}} \right]$$  \hspace{1cm} A6.3
The experiments in Appendix 5 show that best match with experiment was obtained with \( x = 0.42 \) and \( \phi_{\text{min}} / \phi_{\text{max}} = 0.43 \), so that the above equation becomes:

\[
\text{Longitudinal dilution} = 1 - 0.25 \left[ \frac{d_c}{l_c} \right]
\]

**APPENDIX 7: THE WAVELENGTH IN SICHAK’S FACTOR**

Sichak’s factor \( F_S \) is equal to:

\[
F_S = \frac{\pi d_c}{p} \left( \frac{\pi d_c}{\lambda} \right)
\]

The wavelength \( \lambda \) is assumed here to be that down the wire. However it is more convenient to express \( F_S \) in terms of the free-space wavelength \( \lambda_0 \), and so the above equation becomes:

\[
F_S = \frac{\pi d_c}{p} \left( \frac{\pi d_c}{\lambda_0} \right) \frac{c}{V_w}
\]

where \( V_w/c \) is the wire velocity as a proportion of \( c \).

For an air-cored coil the phase velocity down the wire, \( V_w/c \) is less than unity and is given by (see Payne ref 1):

\[
V_w/c \approx 1/k
\]

Where \( k = \sqrt{20/\pi} \left[ \frac{d_c^2}{\lambda_0 p} \right]^{0.25} \)

Therefore \( k = 0.8 F_s^{0.25} \), and thus \( V_w/c \approx 1.25/ F_s^{0.25} \). Inserting this into Equation A7.2 gives:

\[
F_S = 0.74 \left[ \frac{\pi^2 d_c^2}{p \lambda_0} \right]^{1.33}
\]

Where \( p \) is the pitch of the winding
\( d_c \) is the diameter of the winding to the centre of the wire
\( \lambda_0 \) is the free-space wavelength = 300/f for f in MHz

**APPENDIX 8: MEASUREMENT WITH FERRITE**

Appendix 3 describes an experiment with a tube of water formed by inserting a closed plastic tube into a coil and filling around it with water. To test the effect of ferrite, into this inner tube was inserted 7 ferrite rods each having a diameter of 10 mm and a length of 120 mm, equal to that of the coil. The permeability of the rods was not known but inserting them increased the inductance by exactly 4, when measured at a frequency of 0.1MHz (ie at a frequency much lower than the SRF).

The effect of the ferrite was determined by measuring the SRF with the ferrite present and then again with water surrounding it (strictly surrounding the plastic tube into which the rods had been placed). This gave 5.82 and 3.47 MHz respectively, so that the effective dielectric constant was \((5.82/3.47)^2 = 2.8\). From Equation 5.2 Sichak’s factor was calculated as \( F_s = 0.532 \), for \( \sqrt{\mu_{\text{coil}}} = 2 \). This is plotted below along with Equation 3.6.2 for \( \alpha = 4.2 \) and \( \beta =1.1 \) (ie the same as all other experiments):
The measured effective dielectric constant was 2.3% below that predicted from the curve.

One aspect not considered above was the dielectric constant of the ferrite itself. This is likely to have a small effect of the experiment because the two resonant frequencies were measured with this present and it can be assumed that its effect cancelled in the ratio.

**APPENDIX 9 : CALCULATING DIELECTRIC LOSS**

The dielectric constant can be written as:

$$\varepsilon = \varepsilon' - j \varepsilon''$$  \hspace{1cm} A9.1

$\varepsilon'$ is the normally quoted dielectric constant and $\varepsilon''$ is the loss component. Often $\varepsilon''$ is not given in tables of constants, and instead they quote the ‘dissipation factor’ $\tan \delta$ and this is equal to $\varepsilon'' / \varepsilon'$. In RF work it is more convenient to express loss in terms of $Q$, and this is equal to:

$$Q_d = 1 / \tan \delta = \varepsilon' / \varepsilon''$$ \hspace{1cm} A9.2

The $Q$ of most materials which might be used as coil formers is 100 or more. For instance Alumina has a $Q$ of 5000 and glass around 250, depending on the particular glass. Generally the $Q$ is constant over the RF range of frequencies and this will be assumed to be so in the following analysis.

To determine the extra loss produced by the dielectric, we can use the self-capacitance model and calculate the loss from the increase in capacitance due to the dielectric former. The self-capacitance is given by:

$$C = 1 / [(2\pi f)^2 L_0]$$ \hspace{1cm} A9.3

where $f$, is given by Equation 2.2.2

$L_0$ is the low frequency inductance of the coil

When the former is introduced the capacitance will increase by $\sqrt{\varepsilon'_{\text{eff}}}$. If the capacitance with no former is $C_1$ and with a former is $C_2$ then the increase in capacitance is given by

$$C' = C_2 - C_1 = [1 - 1/\varepsilon'_{\text{eff}}] / [(2\pi f)^2 L_0]$$ \hspace{1cm} A9.4
Payne: The Effect of Dielectric Inside an Inductance Coil

where \( f_r \) is the SRF with former
\( L_0 \) is the low frequency inductance of the coil
\( \varepsilon'_{\text{eff}} \) is the effective dielectric constant

The reactance of this parallel capacitance is:

\[
X_c = \frac{1}{(2\pi f_r C')}
\]

A9.5

Where \( f_r \) is the SRF with the former present

The value of the parallel resistance \( R_{dp} \) due to the loss in this capacitance is therefore:

\[
R_{dp} = X_c Q = \left[ \frac{1}{(2\pi f C')} \right] \left[ \frac{\varepsilon'}{\varepsilon''} \right]
\]

A9.6

Inserting the value for \( C' \) from Equation A9.4 gives

\[
R_{dp} = \frac{2\pi f_r^2 L_0}{\left[ f (1 - 1/\varepsilon'_{\text{eff}}) \right]} \frac{\varepsilon' / \varepsilon''}{\varepsilon'_{\text{eff}}}
\]

A9.7

Where \( \varepsilon'_{\text{eff}} \) is the effective dielectric constant
\( \varepsilon' / \varepsilon'' \) is the Q of the dielectric
\( f_r \) is the SRF with the former present

Notice that if the dielectric Q is constant (normally the case) \( R_p \) reduces with frequency.

If this dielectric loss was the only loss then the Q of the inductor would be:

\[
Q_f = R_p / (2\pi f L_m)
\]

A9.8

where \( L_m \) is given by Equation 2.2.1

If the Q due to the conductor losses is \( Q_c \), then the overall Q will be:

\[
Q_t = (Q_f Q_c) / (Q_f + Q_c)
\]

A9.9

Often it is more convenient to express the loss as a series resistance, and the conversion is given by:

\[
R_{ds} = \frac{(R_{dp} X^2)}{(R_{dp}^2 + X^2)}
\]

A9.10

where \( R_{dp} \) is given by Equation A9.7
\( X = (2\pi f L_m) \)
\( L_m = L_o / \left[ 1 - (f / f_r)^2 \right] \)
\( L_o \) is the low-frequency inductance
\( f_r \) is the SRF with the former present.

APPENDIX 10 : CONDUCTOR LOSS

The series resistance due to the conductor, including the effect of resonance, is (ref 1):

\[
R_{ws} = K \sqrt{f} / \left[ 1 - (f / f_r)^2 \right]
\]

A10.1

where \( K = R_{dc} \varphi 0.25 d_w / 0.067 \)
\( R_{dc} = 4 \rho l_w / (\pi d_w^2) \)
Payne: The Effect of Dielectric Inside an Inductance Coil

\[ d_w \text{ is dia of the wire in metres} \]
\[ \rho \text{ is the conductor resistivity (1.77} \times 10^{-8} \text{ for copper}) \]
\[ l_w \text{ is the length of the conductor (straight length) in metres} \]
\[ \phi \text{ is the proximity effect (see ref 1 Appendix 1, and note below).} \]

NB if the ratio of the wire diameter to the pitch is 0.5 (which often gives the highest Q) then \( \phi \approx 1.78 \) for coils with a length equal to twice the diameter or longer.

Sometimes it is more convenient to express this loss as a parallel resistance \( R_{cp} \), and the transformation is given by:

\[ R_{cp} = \frac{R_{cs}^2 + X^2}{R_{cs}} \quad \text{A10.2} \]

where \( R_{cs} \) is given by Equation A10.1

\[ X = (2\pi f L_m) \]
\[ L_m = \frac{L_o}{[1 - (f / f_r)^2]} \]
\[ L_o \text{ is the low-frequency inductance} \]
\[ f_r \text{ is the SRF with the former present.} \]

**APPENDIX 11**: EXPERIMENT TO PROVE LOSS EQUATIONS

A coil was wound similar to Figure A3.2.1, with 59 turns of 1 mm dia wire, onto a thin plastic bottle of outer diameter 49.2 mm, giving a mean wire diameter of 50.2 mm. The length of the winding was 117 mm and the inside diameter of the tube was 47.2 mm. Connection to the VNA was between the ends of the coil. The bottle was filled with distilled water the top of the winding. The water extended by 10 mm beyond the bottom (earthed) end of the coil, but it is shown in Appendix 4 that this will have had an insignificant effect.

Measurements of the series resistance were made with the following results (purple curve):

![Rs v Frequency](image)

*Figure A11.1 Series Resistance including Dielectric Loss*

Also shown is the total predicted series resistance being the sum of Equation A10.1 for the conductor loss (brown) and Equation A9.10 for the series dielectric loss (these are also shown). The error was generally within \( \pm 6\% \) over the whole resistance range of more than 3 decades.

In the predictions the dielectric Q was assumed to reduce as \( 1/f \) (see Appendix 12.2) with a value of 30 at 5.35 MHz, the SRF. This value of Q was measured from the bandwidth of the self-resonance, and assumed to be all due to the dielectric loss since the conductor loss was negligible in comparison at this frequency.
The effective dielectric constant was determined by measuring the SRF’s with and without water and taking the square of the ratio, so the above results do not include any errors in the theoretical prediction of the effective dielectric constant.

APPENDIX 12: DIELECTRICS USED IN EXPERIMENTS

A12.1. Introduction
Three dielectric materials were used in the experiments, water, PTFE and Alumina, and the dielectric properties of these are discussed below.

A12.2. Water
Water is a very useful dielectric for proving the equations because its dielectric constant is very high. However it was found that tap water was very lossy, having a Q of only 1.3 at 5.5 MHz. Distilled water has a much lower loss and so this was used. The dielectric constant of distilled water is often given as 78.2 in the HF range (ref 9). However, it is known that the purity of distilled water can affect its dielectric properties and the water used here was taken from a simple domestic de-humidifier, so its purity was unknown. Consequently the following test was done to determine $\varepsilon'$ and $\varepsilon''$.

The UHF capacitor shown below was immersed in the water and the capacitance measured along with the Q.

![Figure A12.2.1 Capacitor used to determine the dielectric properties of water used.](image_url)

The air capacitance with the vanes fully closed was 12 pf, but not all of this would be affected by the water since some of this capacitance was due to the ceramic insulation. The vanes at minimum capacitance gave 4 pf, and experience had shown that around half of this is due to the ceramic supports. The capacitance which would be affected by the water was therefore estimated as 10 ± 0.5 pf. With water the capacitance increased to 783 pf (averaged over 0.6 to 6 MHz), giving $\varepsilon' = (783-2)/10 = 78.1 ± 5\%$, which is close to the published value.

As for $\varepsilon''$ it was expected that this would follow the Debye characteristic shown below:
The Debye relaxation time for water is about 10 ps at 25°C, giving the peak value of $\varepsilon''$ at around 16 GHz. So at the much lower frequencies used here it was expected that $\varepsilon''$ would be small and decreasing only slightly as the frequency was reduced. However $\varepsilon''$ increased as the frequency was reduced, giving a lower Q at low frequencies as shown below:

The measured curve has the equation $Q = K f$, where $f$ is in MHz, and this is consistent with the above mechanism. $K$ is equal to 3.85 in this case, giving a Q at 5 MHz of 19.4. However it was found that the Q degraded with time, although the proportionality with $f$ did not. As an example water straight from the dehumidifier could have Q as high as 80 at 5 MHz. It was therefore important that the K was determined for each experiment and this was determined from the bandwidth of the SRF resonance. So for instance the Q of the SRF resonance in Appendix 11 was 30 at 5.35 MHz, and so the equation for the dielectric Q used in the equations was $Q = 30 f / 5.35$, where $f$ is in MHz.
A12.3. PTFE

The dielectric constant of pure PTFE is widely reported as 2.1, and constant over a wide frequency range. However it is common to add glass to PTFE to improve its mechanical properties, and this will increase its permeability to 2.4 (for 25% glass). The material used here had been bought as ‘virgin’ and so presumably without a glass filling but to be sure it needed to be measured.

A parallel plate capacitor could have been made by cutting a thin slice of the PTFE from the rod (diameter 15 mm). However the author did not have the tools to cut this accurately, especially the very thin slice which would be necessary to minimize the fringing field around the edges.

The alternative was to measure the resonant frequency of a twin wire transmission line when it was immersed in the dielectric. For this a section of the PTFE rod, 186 mm long, was slotted along its length of two opposing sides to a depth of 4 mm and a copper wire was inserted into these slots and the two wires connected together at one end. The wire needed to have a diameter large enough to be a tight fit in the slots so as to minimize any air gap and the wire used was 1.69 mm over its enamel insulation. The slots were then packed with PTFE tape (plumbers tape). This is shown below:

![Figure A12.3.1 Transmission line within PTFE rod](image)

The wires were connected together at one end and the VNA connected to a tap on each wire, about 10 mm from this end. To contain the field within the PTFE rod it was inserted into a conducting tube, made from copper foil.

The objective now was to measure the $\lambda/4$ resonant frequency, calculate the velocity and compare this with c. In practice it was easier to compare the resonant frequency with that which would be obtained if the velocity was c, and this was $300/(4*0.186) = 403$ MHz, for the length of 0.186 meters. Resonance was measured at 281.2 MHz so the effective dielectric constant was $(403/281.2)^2 = 2.06$. However this assumes that there is no end effect (whereby the fields extend beyond the end making the wires appear longer). To test this a copper sheet was placed very close to the end of the dielectric (about 0.5 mm away) and the resonant frequency reduced by less than 1%. So it would appear that the interface with the air at the end provided a very good reflection to the waves.

However the resonant frequency was sensitive to hand proximity and so some field was leaving the sides. To contain this a copper tube of length 0.16 metres was wrapped around the PTFE and its joint soldered. It was positioned so that one end was flush with the end of the PTFE away from the VNA connections. The resonant frequency dropped to 271.9 MHz, giving $\varepsilon_r = 2.2$. 

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The actual dielectric constant will be greater than this because there is air space around the wires (they are circular in rectangular slots), and the PTFE tape filling the outer part of the slots had air gaps within it. So it would be reasonable to assume that the dielectric constant was 2.3 ±0.1.

A12.4. Alumina

A tube of Alumina was bought having the following specification:
99% Purity Aluminium Oxide (Alumina). Balance is Silicon Dioxide
Fired to full density so not porous
OD : 6.35 ±0.33 mm (measured by author at 6.25 ±0.1 mm )
ID  : 4.75 ±0.33 mm (measured by author at 4.8 ±0.1 mm )
Wall thickness : 0.75 mm average
The supplier did not give the dielectric constant but Reference 11 gives 9.8 for 99.5% at 1MHz, 9 for 96%, and 9.1 for 94% purity, and assuming a linear increase in dielectric constant versus purity, for 99% purity $\varepsilon_r = 9+0.8*3/3.5 = 9.68$. 
REFERENCES

1. PAYNE A N : ‘Self-Resonance in Coils’, http://g3rbj.co.uk/
8. PAYNE A N : ‘The Inductance of Ferrite Rod Antennas’, http://g3rbj.co.uk/

Issue 1 : November 2015

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