

SKIN EFFECT : ELECTROMAGNETIC WAVE OR DIFFUSION ?

At high frequencies current in a conductor flows mainly on its surface, and this is known as the skin effect. Two possible mechanisms are given in the published literature, the first being that it is due to penetration of the conductor by an electromagnetic wave, and the second is that it is due to diffusion. This article resolves the issue by experiment, and shows that diffusion is the correct explanation. This has important implications in the screening of electronic circuits since many authorities in this area assume that the conductor is penetrated by an EM wave.

1. INTRODUCTION

It is well known that at high frequencies current in a conductor flows mainly on its surface. This is called the skin effect, and is often explained as the penetration of the conductor by an electromagnetic wave which then flows within the conductor. It is assumed in some articles on screening (eg ref 1&2) that when this wave reaches the opposite face and interfaces with air, a portion of the wave will be reflected back into the conductor. However it is shown here that reflection within the conductor does not take place and that the skin effect is due to diffusion and not to penetration by an EM wave.

Interestingly the equations derived from the EM analysis for the attenuation and phase shift with depth into the conductor also apply to the diffusion. Most textbooks on electromagnetics show the derivation of these equations, but the author has been unable to find any well quantified experiments which would prove them and this led to the experiment described here.

2. THE SCREENING PROBLEM

Metal screens are used to minimize interference between electronic systems, and they do this by a combination of reflection from the outside surface and attenuation through the thickness of the conductor. It is important therefore to be able to quantify the attenuation through the conductor, here defined as the ratio of the surface current on the excited side to the surface current on the opposite side.

It is well established that in a wide thick flat conductor, the current density on the surface, J_0 , decreases exponentially with depth according to the following equation (ref 3):

$$J_z = J_0 e^{-z/\delta} \quad \text{where} \quad \delta = [\rho/(\pi f \mu)]^{0.5} \quad 2.1$$

$\mu = \mu_r \mu_0$
 μ_r is the material relative permeability
 $\mu_0 = 4\pi \cdot 10^{-7}$
 ρ = resistivity (ohm-metres)

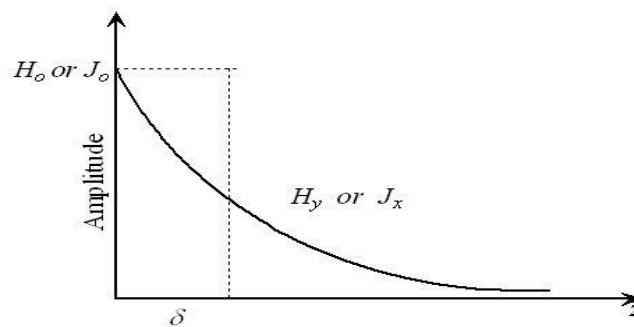


Figure 2.2.1 Current density at high frequencies

When the conductor thickness is infinite, the area under the above curve has the same area as that of a current uniformly distributed down to depth of δ and zero at greater depths (see dotted lines above), and this leads to the definition of skin depth :

$$\text{Skin depth } \delta = [\rho/(\pi f \mu)]^{0.5} \quad 2.2$$

Associated with this exponential reduction in amplitude is a change of phase, with an angle of one radian at one skin depth. So the phase angle θ at a depth x is given by :

$$\begin{aligned} \theta &= x / \delta \text{ radians} \\ &= 360 x / (2\pi \delta) \text{ degrees} \end{aligned} \quad 2.3$$

where δ is given by Equation 2.2

The penetration of current into the conductor thus has a wave-like characteristic and indeed it is often described as the penetration of an EM wave into the conductor. However there is an alternative view, and Spreen (ref 4) shows that the above equations can also describe diffusion, which is defined as the net movement of a substance from a region of high concentration to a region of low concentration. Given that conduction in metals is a movement of charged particles (electrons), diffusion seems to be a more likely mechanism.

To understand the important difference between these two mechanisms consider a flat plane conductor with an EM wave starting at one face, traversing the thickness of the conductor and reaching the opposite face. At this surface there is a major discontinuity with the air, and the wave will be reflected. Given that the wave impedance in the metal, E/H will be around $1/100^{\text{th}}$ that in the air (ref 5) the situation is similar to a transmission-line terminated in a very high impedance, so that the forward current at the surface will be nearly cancelled by that of the return wave. So on this assumption of an EM wave in the conductor this reflection will give a very small surface current and add considerably to the screening effect, probably of the order of 20 dB or more. Alternatively if diffusion is the mechanism there will be no reflection and there will be a much higher current at the opposite surface, and a much lower screening effect.

3. EXPERIMENTAL RESULTS

3.1. Introduction

The above discussion assuming an EM wave showed that the current on the opposite surface could be greatly reduced by reflection at the air interface. However there is an alternative view and Wheeler (ref 6) assumes that reflection produces a doubling of the current at the surface, thereby reducing the screening effect by 6dB.

So there are three possibilities which need to be tested against experiment:

- a) that the current on the far surface is due to diffusion, and is therefore given by Equations 2.1 and 2.3,
- b) that the surface current is due to an EM wave and reflection considerably reduces the surface current given by those equations,
- c) reflection of the EM wave doubles the current on the far surface.

This section gives a brief outline of the experimental technique and the results obtained. The details are given in Appendix 1.

3.2. Measurement technique

A high frequency current was injected into the surface of a copper foil having dimensions of 350 mm x 300mm and a thickness 0.055 mm. Measurements were then made of the current on the opposite surface, both its magnitude and phase, and compared with the theory given by Equations 2.1 and 2.3.

3.3. Results

The following graphs show the measured values of attenuation and the theoretical values given by Equation 2.1.

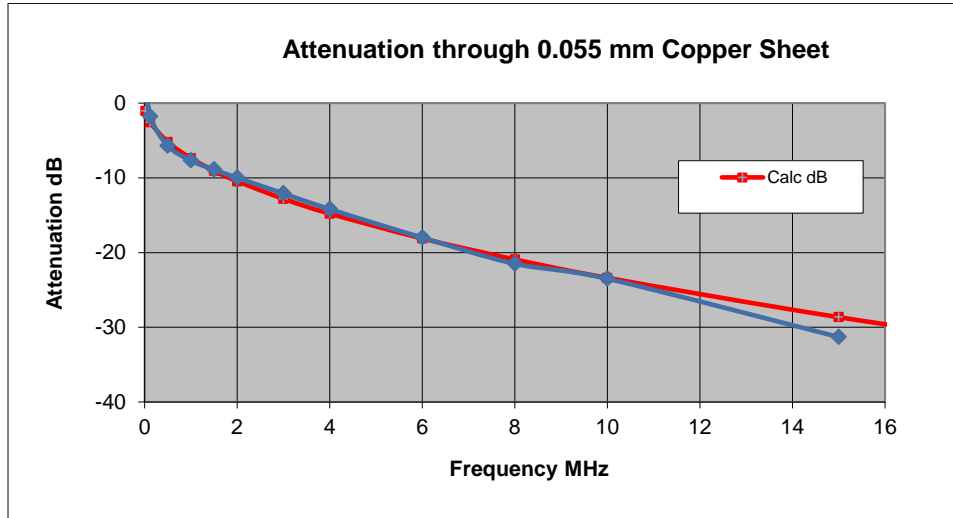


Figure 3.3.1 Attenuation through a Copper Sheet of thickness 0.055 mm

The correlation is very good and shows that the mechanism for current penetration in conductors is by diffusion, and that no reflection takes place at the metal/air interface.

The measurements are possibly more informative if the x axis is the ratio of the foil thickness to that of the skin depth, t/δ , rather than the frequency as below:

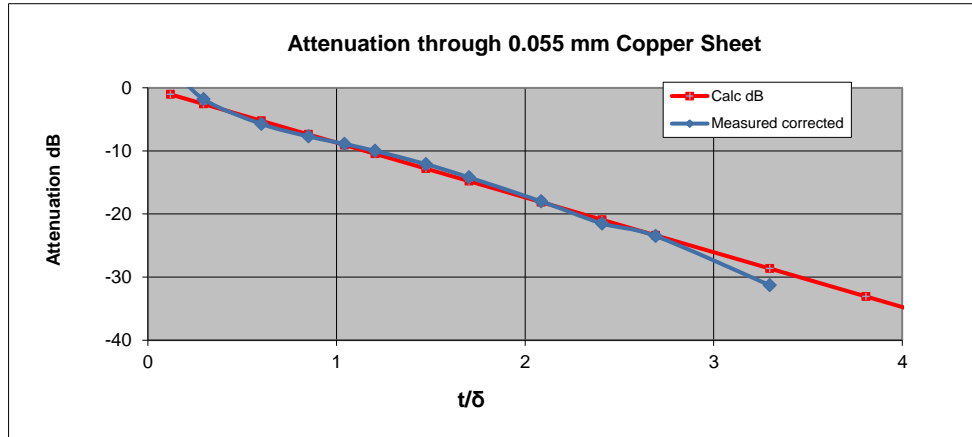


Figure 3.3.2 Attenuation through a Copper Sheet of thickness 0.055 mm

For the calculated skin depth it was assumed that the foil was pure copper and thus had a resistivity of $1.68 \times 10^{-8} \Omega\text{m}$ at 20°C , and a permeability of unity. The actual foil used will inevitably have some impurities but it is shown in Section 5.6 that these will not significantly affect the value of the skin depth.

Below is shown the measurement of phase shift through the foil, compared with that calculated from Equation 2.3

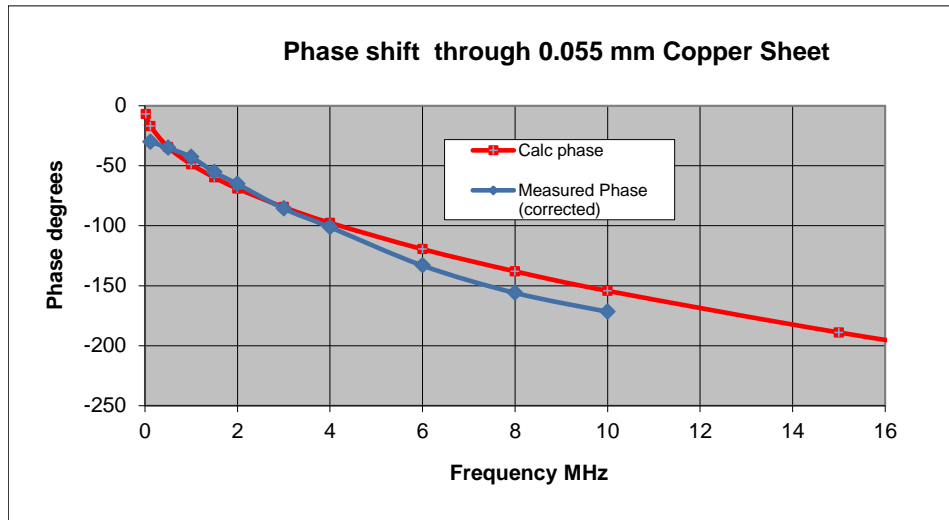


Figure 3.3.3 Phase shift through a Copper Sheet of thickness 0.055 mm

The correlation is reasonable over much of the range but at higher frequencies there is a phase offset of 16° . This is unexplained but is likely to be due to experimental error, in particular the poor signal to noise ratio in the VNA (see Appendix 1).

It is interesting to note that at about 14MHz current on the far surface of the foil has a phase shift of 180° , so that it is actually flowing in the opposite direction to that on the excited surface.

4. DISCUSSION

Given that diffusion is the mechanism it raises the question as to *what* is diffusing? The experiment here measured currents and so clearly there is a diffusion of moving charges (electrons) into the conductor. However it can be shown that there is also magnetic diffusion, and reference 13 shows just such an experiment. Given that the magnetic field is due to the movement of charges, diffusion of the field is not surprising.

Although it is shown here that current propagates by diffusion it is likely that there *is* some penetration by an electromagnetic wave. However this will require displacement current in the conductor and this will be exceedingly small compared with the conduction current. Reference 5 gives the ratio of these currents as $\mathcal{D} = \sigma / (\omega\epsilon)$ where σ is the conductivity, and for copper at 1MHz this ratio is approximately 10^{12} !

The experiment here has considered skin effect in a plane conductor, but skin effect also arises in round wires carrying current. The emf driving the current is then connected between the ends of the wire and not on its outside surface, so current will presumably diffuse along the *length* of the wire, rather than into its diameter. However diffusion is a very slow process with a velocity of $v_p = \omega\delta$, (ref 5) and in copper this is equal to 406 m/s at 1 MHz, much lower than the actual velocity which is close to $c = 3 \cdot 10^8$ m/s . So clearly current does not diffuse down its length. Edwards and Saha (ref 12) give the following explanation : ‘..... the currents needed to energise electrical loads initially propagate along the outside of the cable (transmission line) to the load as displacement currents in the insulation at velocities approaching c . The displacement current builds up the line current on the surface of the conducting cables by multiple reflections, and this current diffuses into the interior of the conductor. If it were not for the displacement current energy transmission via copper conductors would be virtually impossible because of the long diffusion times and attenuation’.

5. CONCLUSION

The experiment described here shows that there is no discernable reflection at the metal/air interface, and therefore that the skin effect is due to diffusion rather than penetration by an electromagnetic wave.

6. Appendix 1 : Details of Experiments

6.1. Introduction

The objective was to set-up a high frequency current on one face of a sheet of copper foil, and to measure the current on the opposite face, both its magnitude and phase.

One side of the copper foil is shown below.

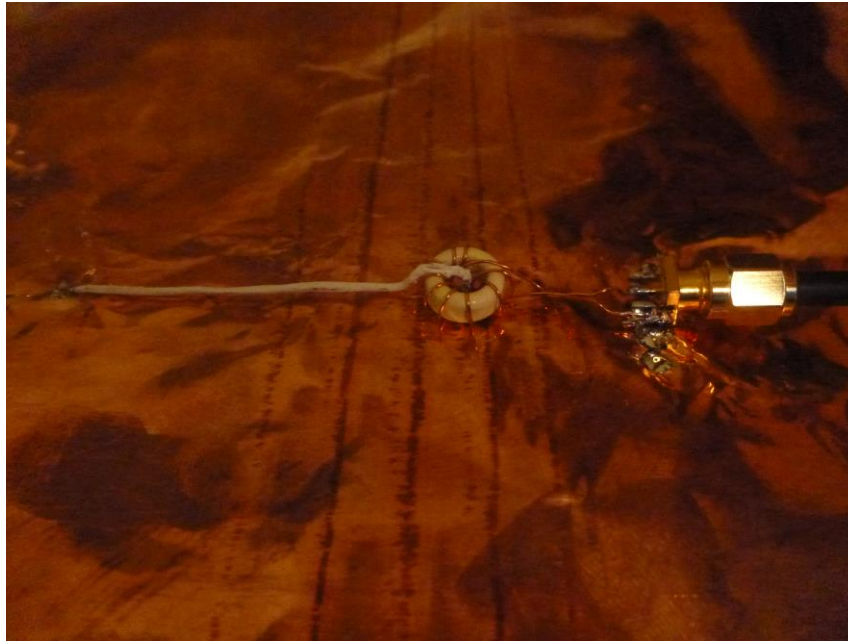


Figure 5.1 Photograph of one side of the metal foil

Shown in this photograph, from right to left, is the SMA connector to the current source, a transformer to improve the match, and a wire lead (painted white for clarity) taking the current to two points on the surface of the foil. One of these soldered contact points is visible on the far left and the other is hidden by the transformer. This wire and its connections are subsequently referred-to as the ‘current probes’.

A similar arrangement is on the other side of the foil, with its current probe slightly shorter so that its contact points with the foil are not directly below those shown. The output from the lower transformer is taken to a signal detector. In practice the signal source and the detector were one unit : an Array Solutions, UHF Vector Network Analyser.

6.2. Current probes (one coloured white in above photo)

Current was injected by a current probe consisting of a 76 mm length of 20 swg copper wire, soldered to the foil at both ends. The detection probe was located directly below, but was slightly shorter at 52 mm to minimise any local effects around the injection points. It was found that both wires needed to be kept close to the foil to minimise the inductance.

6.3. Transformers

The current probes have a very low impedance compared to that of the VNA, and indeed they are almost a short circuit. To provide a better match two transformers were constructed, using Ferroxcube toroidal core TX 10/6/4-3E5, wound with 10 turns of 0.4 mm copper wire. These windings were evenly distributed around the toroid.

The frequency response of the transformers and current probes was measured using the following set-up :

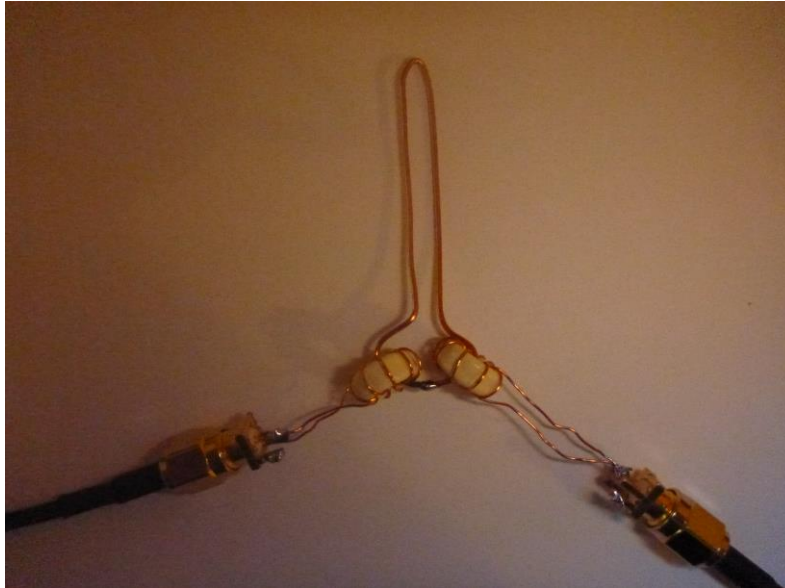


Figure 5.3.1 Calibration set-up for Transformers and Current Probes

The transformers are connected ‘back to back’ via a hairpin loop which simulates the two current probes, and their inductance. Of course this arrangement leaves out the copper foil, which would have a shorting effect and increase the loss considerably, but it was assumed that this omission would merely give an offset in loss but would not affect the change of loss with frequency. This offset is discussed later.

A limitation of the above is that the hairpin loop simulates current probes with the same length, whereas they were in fact slightly different lengths.

The measured loss and phase response is shown below.

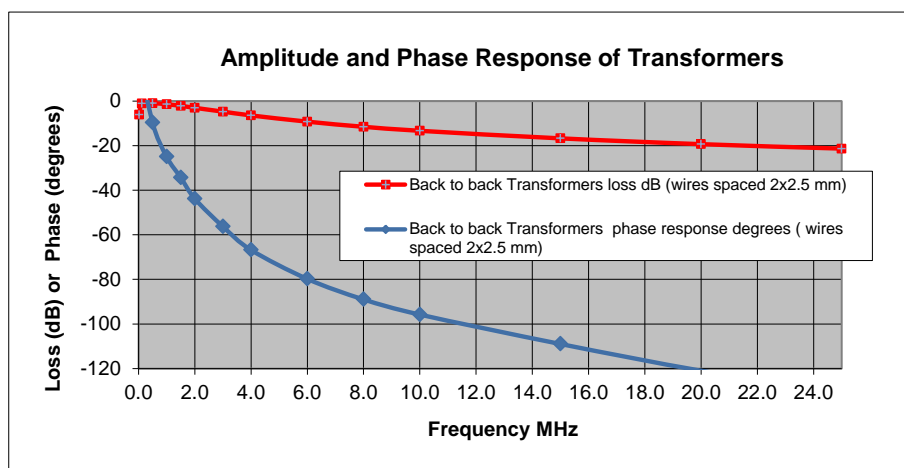


Figure 5.3.2 Loss and Phase of back-to-back transformers and probes

The transformers needed to have a very wide frequency range from 0.12 to 15 MHz and so some compromise was necessary resulting in a high loss at high frequencies. These measurements were used to correct the skin depth measurements.

An attempt was made to include the shorting effect of the foil in the calibration while not including the attenuation with depth. For this the hairpin loop above was threaded through two holes in the foil and soldered to one side of each hole. It was hoped that current would pass from one side of the loop to the other unchanged except that a portion would be diverted into the foil. This method of calibration was partially successful as the following curves show ;

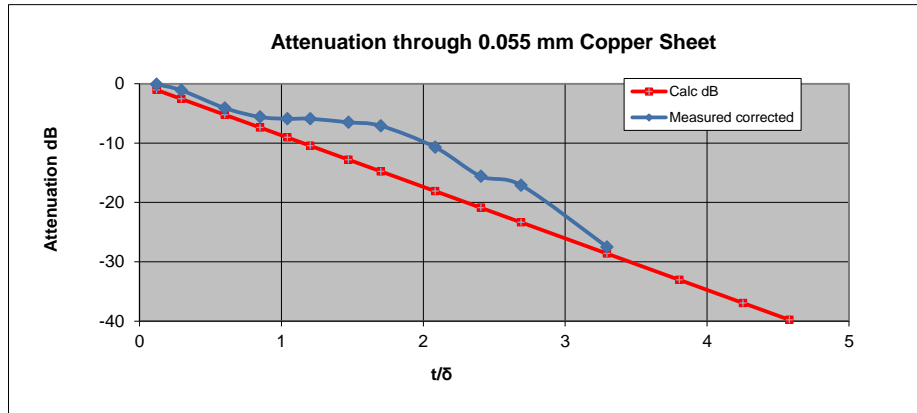


Figure 5.3.3 Attenuation results with Foil in the probe calibration

In the above the measured values were corrected only by the ‘with foil’ probe calibration and no offset was used. It is seen that at small values of t/δ the results were within about 2 dB of the calculations, but deviates at higher values (ie higher frequencies) but the reason is not known. However the good correlation at low frequencies supports the results presented in Figures 3.3.1 and 3.3.2.

6.4. Attenuation Offset

In the transformer tests described earlier, the copper foil was not present in the test set-up (its skin effect would have confused the measurement). However when the foil is introduced there is a large offset loss because it nearly shorts-out the probes, and it is necessary to determine the value of this loss. This can be done because at very low frequencies the attenuation through the copper is very close to zero, and so an offset can be chosen to make this so, and then applied to all frequencies. The lowest frequency used in the measurements was 0.12 MHz, and this was not as low as would be ideal for determining this offset because the attenuation at this frequency was theoretically 2.8 dB, rather higher than ideal. So it was assumed that the measurements were correct at this frequency and the offset set to make the measured value 2.8 dB, and for this an offset value of 48 dB was required.

The impedance of the foil between the two probe points will vary with frequency and this will produce an error since this is not included in the measurements. However the impedance of the probes will be very much higher than the foil so they will appear as a constant current source to the foil and therefore the injection current will be essentially independent of the foil impedance.

6.5. Position of connecting leads

The connection leads need to be located at positions of minimum current density to have minimum effect on the measurements. Contours of constant current are shown below, and the minimum density is to the right of source r_2 and to the left of the source r_1 .

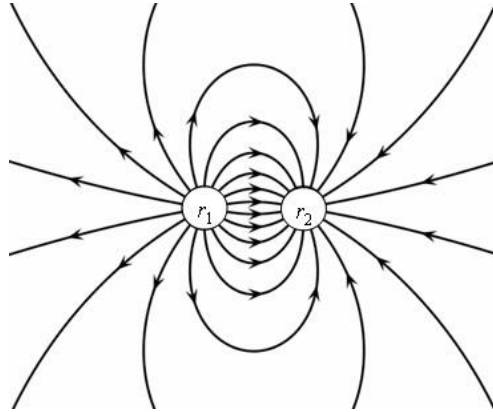


Figure 6.5.1 Contours of Constant Current

6.6. Impurities in the Copper Foil

The foil tested is likely to be of standard commercial grade and so is assumed to have a purity of 99.9%. So impurities amount to only 0.1%, but nevertheless can have an effect on the resistivity and permeability. No information has been found on the concentrations of trace elements but some guidance is given by reference 10 which gives concentrations for very high purity copper of 99.96%. If these are scaled to give 99.9% purity this gives :

	Concentration %
As	0.0033
Bi	0.0033
Fe	0.0120
Ni	0.0433
Pb	0.0208
Sb	0.0045
Sn	0.0043
Zn	0.0053

Permeability

Assuming that pure copper has a relative permeability of unity then the overall permeability with trace elements, μ_{RT} , will be :

$$\mu_{RT} = 1 + (\mu_{R1} - 1) C_1 + (\mu_{R2} - 1) C_2 + \dots \quad 6.6.1$$

where μ_{R1} and C_1 are the permeability and concentration of impurity 1
 μ_{R2} and C_2 are the permeability and concentration of impurity 2
 etc

Most elements will not affect the overall permeability because their own permeability is close to unity. The exceptions are nickel with a permeability of 110, and iron with 400 (there is some uncertainty here and for instance ref 8 gives a range of 300-500 for its initial permeability). So from the above equation the overall permeability will be :

$$\mu_{RT} = 1 + (110 - 1) * 0.043 / 100 + (400 - 1) * 0.012 / 100 = 1.1 \quad 6.6.2$$

Resistivity

The resistivity will be increased by all the trace elements with some such as iron having a large effect and others having a small effect. Reference 9 (figure 1) gives curves for the resistivity increase for the various impurities, and these curves can be represented by the following equation:

$$\rho_t = [1.68 + C_1 (\Delta R_1) + C_2 (\Delta R_2) + C_3 (\Delta R_3) \dots\dots\dots] 10^{-8} \quad 6.6.3$$

where 1.68 is the resistivity of pure copper
C₁ is the concentration of first impurity in %
ΔR₁ is the slope of the curve for the first impurity
etc

From the curves ref 9 :
ΔR for P = 14.3
ΔR for Fe = 9.5
ΔR for Si = 6.3
ΔR for As = 5.45
ΔR for Cr = 4.2
ΔR for Mn = 2.8
ΔR for Ni = 0.7

So from the above equation the overall resistivity for impurities of Fe, As, Ni :

$$\begin{aligned} \rho_t &= [1.68+0.012*9.5+0.0033*5.45+0.043*0.7] 10^{-8} \\ &= [1.68+0.114+0.018+0.03] 10^{-8} \quad = 1.84 10^{-8} \end{aligned} \quad 5.5.4$$

This is an increase of 9.6% over pure copper.

Summary

The assumed impurities increase the permeability by 10% and the resistivity by 9.6%, so the skin depth Equation 2.2, is virtually the same as for pure copper (ie within 2%).

6.7. Soldering

The probes were soldered to the foil, but it was found that excessive soldering could affect the results. In a test the frequency response was measured (such as figure 3.3.1) and then the two solder connections to one of the current loops was re-heated for about 5 secs each, and the response re-measured after the connection had cooled. The response had reduced by 2.5 dB across the frequency range. This reduction is probably due to diffusion of copper into the solder, and reference 7 indicates that this diffusion could have a depth of 10 μm, or around 20 % of the foil thickness. Solder has about 10 times the resistivity of copper. It was therefore important that the solder joints were made as quickly as possible, and not re-soldered.

6.8. Thickness of Foil

The foil thickness was measured with a standard micrometer as 0.06 mm. However this instrument is likely to have an error of up to ±0.01 mm and so the actual thickness could be between 0.05 and 0.07 mm. The error can be reduced by measuring two thicknesses of foil and this gave 0.11 ± 0.01 mm giving a single thickness of 0.055 ± 0.005 mm, and this was the thickness used in the calculations

6.9. Dynamic range and Signal to Noise ratio

The signal level on the far side of the foil is very small, and at the high frequencies can be 85dB below the incident power. The noise floor of the VNA was not much lower than this and so the measurements at the higher frequencies are subject to noise especially the phase measurements which seemed to be more susceptible to poor signal to noise ratio.

REFERENCES

1. CHRISTOPOULOS C : 'Principles and Techniques of Electromagnetic Compatibility', 2nd Edition, CRC Press, 2007 (see pages 156-161)
2. SCHELKUNOFF S A ' Electromagnetic Waves' 6th Printing, D Van Nostrand Co Inc, 1948, (page 304)
3. PAYNE A N : 'Skin Effect, Proximity Effect and the Resistance of Rectangular Conductors', <http://g3rbj.co.uk/>
4. SPREEN J H : 'A Circuit Approach to Teaching Skin Effect' <http://ilin.asee.org/Conference2007program/Papers/Conference%20Papers/Session%201A/Spreen.pdf>
5. LORRAIN P, CORSON D P, LORRAIN F : 'Electromagnetic Fields and Waves', Freeman and Company, 3rd Edition, 1988, page 540.
6. WHEELER H A : 'Formulas for the Skin Effect', Proc. IRE, September 1942 p412-424.
7. Di MAIO D, HUNT C. P, and WILLIS B : 'Reduced Copper Dissolution in Lead-free Assembly', NPL Good Practice Guide, No 110. http://publications.npl.co.uk/npl_web/pdf/mgpg110.pdf
8. AK STEEL INTERNATIONAL : <http://www.pctmg.nl/uploads/API-Brochure.pdf>
9. COPPER DEVELOPMENT ASSOCIATION : <http://www.copper.org/publications/newsletters/innovations/1997/12/wiremetallurgy.html>
10. DOIDGE P S : 'Determination of Trace Impurities in High-Purity Copper by Sequential ICP-OES with Axial Viewing', Agilent Technologies Application Note, 1998, <http://www.chem.agilent.com/Library/applications/ICPES-25.pdf>
11. Permeability of Materials : http://en.wikipedia.org/wiki/Permeability_%28electromagnetism%29
12. EDWARDS J & SAHA TK : 'Diffusion of Currents Into Conductors', <http://espace.library.uq.edu.au/view/UQ:9820/aupec-03-6.pdf>
13. MIT Electromagnetic Field demonstration : <https://www.youtube.com/watch?v=XNDk5YcycVM>