LITZ CABLE FOR HF SINGLE LAYER COILS

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Litz cable can have a lower ac resistance than solid wire of the same overall diameter, but conventional wisdom says that it loses this advantage above a frequency of about 1 MHz. However it is shown here that Litz can have the lower resistance up to at least 14 MHz with readily available cable.

1. INTRODUCTION

The effective resistance of conductors to radio frequencies is considerably more than the ohmic resistance measured with direct currents. This is because of the action known as skin effect which causes the current to be concentrated in certain parts of the conductor, and leaves the remainder to contribute little or nothing toward carrying current. In particular in a conductor of circular cross-section the current concentrates in a thin 'skin' around the circumference and the centre of the conductor carries little if any current.

Skin effect arises because of the magnetic field which the conductor produces around itself, but if this field also intercepts other conductors this will also increase its resistance. In particular if the wire is wound into a coil, each turn of the wire induces loss-making eddy currents into adjacent turns, indeed even of turns some distance away in that it contributes to an overall field down the coil. The power lost in these eddy currents must come from the wires responsible for the magnetic field, and so these wires have an apparent increase in their resistance. In addition, as flux passes down the coil only a part of it reaches the end, and a proportion leaks away into the wire, particularly towards the ends of the coil, and this sets up further eddy currents. The eddy currents which are induced in the wires increase the resistance and this is known as the proximity effect.

One way to reduce the skin effect and proximity effect is to form the conductor from a large number of small wires, insulated from each other, thoroughly interwoven and connected in parallel only at their ends. If the stranding is properly done, each strand will on the average link with the same number of flux lines as every other, and the current will divide evenly between strands. If the diameter of each strand is small, it will have relatively little skin effect over its cross-section, with the effect that all the material is effective in carrying the current and a radio frequency resistance approaching the direct current resistance results.

Stranded cable can therefore reduce the resistance to alternating currents, and conventional wisdom is that this advantage disappears at frequencies above about 2 MHz, but it is shown here that it can extend to much higher frequencies.

Key equations are highlighted in red.

2. STRANDED CABLES

2.1. Introduction

Stranded cable can give significantly lower losses if the diameter of strands is small compared with the skin depth (i.e. d_s/δ is small), the bundle of strands is twisted and the individual strands are insulated from one another except at the ends where they are connected together. In addition most authors say that the strands should be interwoven in such a way that each strand occupies all possible positions in the cable, so that each on average is subjected to the same magnetic field over its length. However, Welsby (ref 5, p74) says that this is necessary only if one wants to minimise the resistance of a *straight* cable, where the eddy loss is due to the currents in adjacent strands. But when the cable is formed into a close wound coil the main loss is due to the proximity of adjacent wires and Welsby says 'It can be shown that, for coils with more than say 10 closely wound turns, this form of transposition is not necessary at all, a plain axial twist of the whole wire being all that is required' (see also Section 2.7).

In this report the bundle of strands is called a cable, and a cable with 7 strands is shown in Figure 2.1. Cables with from 5 strands to 100's of strands are readily available.

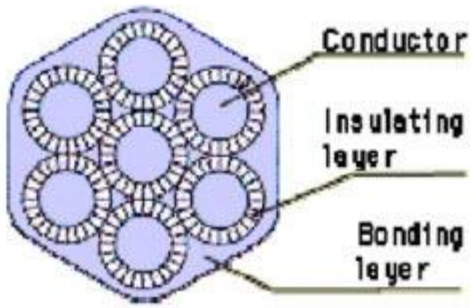

Figure 2.1 Cable with 7 Insulated Strands

The strand diameter is designated d_s, and the overall diameter of the n strands is the bundle diameter d_o. Insulating the whole bundle is a covering often of silk or nylon, but it is not always possible to distinguish these materials and so in following text it is referred-to as silk. The diameter over this insulation is the cable diameter d_{cable}.
Stranded cable is often designated as, say, 98/38 which means 98 strands of 38 gauge wire. This can be confusing because there are two commonly used wire gauges - AWG (American Wire Gauge) and SWG (Standard Wire Gauge), the British standard. These give very different diameters and for 38 gauge give 0.101 mm and 0.1524 mm respectively.
To avoid this confusion Litz wire is best described by the number of strands and the diameter of each strand in mm e.g. 98/0.0101, and this is used here except when quoting past papers when the designation used by their authors is used.

2.2. Cable Types

Mere subdivision into strands is not sufficient to reduce the ac resistance and in fact if the strands are parallel and untwisted the bundle will have the same resistance as solid wire of the cross-sectional area of copper. If such a cable is used as a straight conductor a high frequency current will penetrate the outer strands only and little will flow in the centre strand, similar to skin effect in solid wire. If the cable is now wound into a coil the cable is cut by axial and radial magnetic flux (see Payne ref 1), and both will induce loss making currents. In particular the axial flux will enclose the strands on the outside of the coil more than the inner ones and so the outer strands will carry little current. This is the situation with normal solid wire where the current flows mainly on the side of the wire closest to the coil axis, and so again, when wound, this stranded cable with no twist has no advantage over solid wire.
On the other hand if the cable is *twisted* down its length, and then wound into a coil the magnetic flux will now enclose all strands equally (on average), and so the resistance will reduce. The most obvious effect of this twisting then is that current flows on the whole circumference of the wire, rather than being concentrated towards the axis of the coil, as it is with solid wire. However notice that twisting will not improve the resistance of the straight conductor (i.e. when not wound into a coil), and this is an important difference which has confused some discussions on the effectiveness of stranded cables (see paragraph 2.7).
If there is a central strand as in Figure 2.1 this will carry little current, because it is screened by the eddy currents in the surrounding strands (i.e. skin effect). However it has an adverse effect because it is connected to the other strands at each end where the induced voltage in these strands will send a *reverse* current down the central strand. This central strand therefore not only does not contribute to conduction but actually leads to higher losses. This problem can be reduced if the cable is both twisted *and* the strands are so interwoven that each strand occupies all possible positions from the centre of the bundle to the outside. Then all strands will have the same induced voltage, and will carry the same current whether the cable is used in isolation or in a coil. This type of cable is known as Litz from the word Litzendraht, which is

German for braided or woven wire, but unfortunately the term Litz is often used to describe any stranded cable.
In addition to the above cables of insulated and twisted, and insulated, twisted and woven there is a third possibility, and that is a cable consisting of twisted strands which are *not* insulated. This has the advantage of low cost, and it also has a greater area of copper for a given bundle diameter, because space is not taken with insulation. Despite not having intentional insulation the slight oxidation of the surface of each strand makes a current path between strands more resistive than a path down the strand. Cable of this type has been investigated recently by Tang & Sullivan (ref 2) for transformers, but its application to inductance coils has not been investigated.

2.3. Bundle Diameter d_o

The bundle diameter d_o is defined as that of the smallest circle which will just enclose the n strands, including their individual insulation. It does not include any insulation of the bundle as a whole.
Equating the cross-sectional areas gives:

Bundle diameter $d_o = [(d_{si})^2 n / \eta]^{0.5}$ **2.3.1**

where d_{si} **is the diameter of the strand including its insulation**
n is the number of strands
η is the packing density

The packing density η depends on the number of strands and below in blue is shown the mathematical maximum density (see Wiki : Packing density ref 3) :

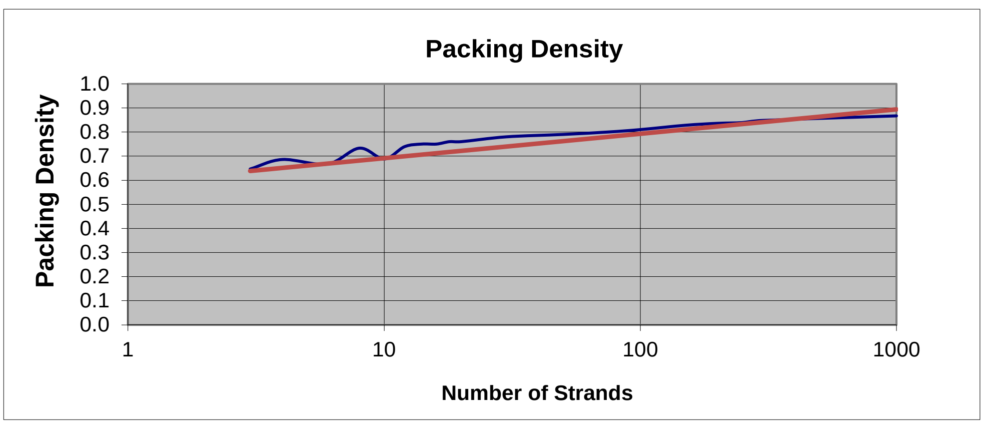

Figure 2.3.1 Packing Density

Note that some numbers of strands do not pack well, particularly when n is small and this accounts for the ‘ripples’ at low values on n.
The practical packing density is likely to be less than this especially when the number of strands is small, and in red is the following empirical curve :

$$\eta = 0.044 \text{ Ln } (n) + 0.59 \qquad \textbf{2.3.2}$$

This curve is 2% below the theoretical optimum when n =150, and 5% below when n=20.

Also needed in Equation 2.3.1 is the diameter of the strand over its insulation. This insulation is traditionally enamel, and is usually described as such, but is more likely to be polyimide. The thickness of this insulation depends upon the diameter of the strand, and Sullivan (ref 4) gives the following empirical equation derived from a study of actual cables, for strand diameters from 0.007 mm 0.25 mm :

$$d_{si} = 0.08\ \alpha\ (d_s/0.08)^{\beta} \qquad 2.3.3$$

where d_s is the diameter of the conductor

(the significance of the 0.08 figure is that it is the reference diameter for determining the equation)

For single-build insulation α=1.12 and β=0.97, and for heavy build insulation α=1.24 and β=0.94. This equation is plotted below as the ratio of copper area to total area over insulation, for wire diameters from 0.01 mm to 0.26 mm :

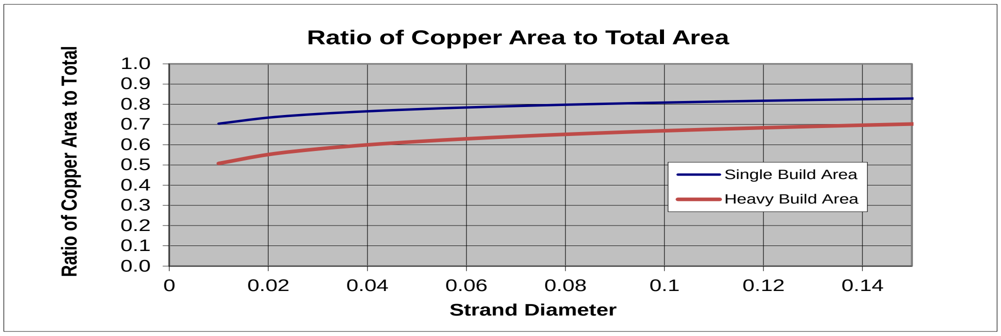

Figure 2.3.2 Ratio of Strand copper area to area with insulation

For the evaluation later the mean of these taken, so α=1.18 and β=0.95, unless otherwise stated

It can be seen that the strand insulation considerably reduces the area for conduction and has encouraged the investigation of un-insulated strands.

2.4. Cable Diameter d_{cable}

The cable diameter d_{cable} is that of the bundle plus the thickness of the overall silk insulation so that the maximum achievable winding ratio d_o/p, (where p is the pitch) is given by d_o / (d_o+ 2t), where t is the thickness of the silk. Manufacturers generally give the thickness in the range 0.03 - 0.05 mm giving :

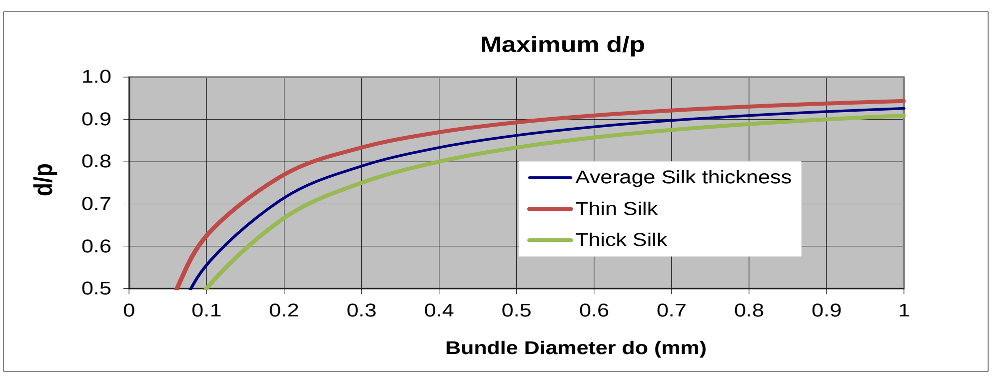

Figure 2.4.1 Maximum d_o/p due to Silk Insulation

Taking the average thickness the cable diameter is assumed to be :

$$\mathbf{d_{cable} = d_o + 0.08\ mm} \qquad \textbf{2.4.1}$$

The above shows that the overall silk insulation can considerably reduce the maximum d_o/p which can be wound, and for the Litz cables measured here with bundle diameters of around 0.5 mm the largest achievable d_o/p is 0.86.

2.5. Strand Diameter d_s

The strand diameter d_s should be small compared to the skin depth (Appendix 2), to ensure that the whole cross-section of each strand carries current, and then the resistance will be not much higher than the dc resistance.
When the frequency is raised so that the strand diameter becomes equal to about 1.5 skin depths the cable loses its advantage over solid wire and this happens at approximately the following frequencies (approximately because other factors have an effect, such as winding ratio, number of strands etc.) :

Strand Dia (mm)	Cross-over (MHz)
0.1	0.95
0.07	1.9
0.04	5.8
0.025	15

Notice that if the strand diameter is halved the cross-over frequency increases by a factor of 4 because the skin depth is proportional to $1/\sqrt{f}$.

2.6. Strand Length

The length of the strands is slightly longer than the length of the cable because they follow the helical path of the twist. The ratio of strand length to bundle length is given by:

$$l_s / l_b = [\, l^2_b + (\, n\, \pi\, d_t)^2]^{0.5} \qquad 2.5.1$$

where d_t is the average diameter of the twist helix
n is the number of twists down the length
l_s and l_b are the strand length and bundle length respectively

A key factor in the above equation is the average diameter of the twist helix d_t. Its maximum value is clearly that of the bundle, but as the strand weaves through the bundle at the middle the helix diameter will be zero. It is assumed here that the average value is half the bundle diameter $d_o/2$. The standard twist pitch of commercially available cable is between 10mm and 20 mm (see also paragraph 2.7) so taking the average value this is equal to 67 twists per metre, so the above equation becomes :

$$\mathbf{l_s / l_b = [\, 1 + (67\, \pi\, d_o/2)^2\,]^{0.5}} \qquad \textbf{2.5.2}$$

For the cables evaluated here this ratio ranges from 1.001 for a small number of strands of small diameter (11 of 0.04 mm diameter), to 1.05 for a large number of strands of larger diameter (256 of 0.07 mm). So this increase in strand length is not always negligible and has been included in the evaluation.

2.7. Number of Twists

It has been stated that the strands should be twisted to reduce the effects of the axial and radial magnetic fields generated by the coil, and the following analysis is based on that by Welsby (ref 5).
Figure 2.7.1 represents the cross-section of a cable in which the strands are evenly twisted about the axis of the bundle, with two typical strands represented by the sinusoidal curves. It is important to note that these strands are connected together at the ends (as are all strands), so that a circulating current can flow between

them. It is assumed here that no current is fed through the wire as whole, but that the cable is cut by a perpendicular magnetic field due to the axial and radial fields of the coil (see Payne ref 1).

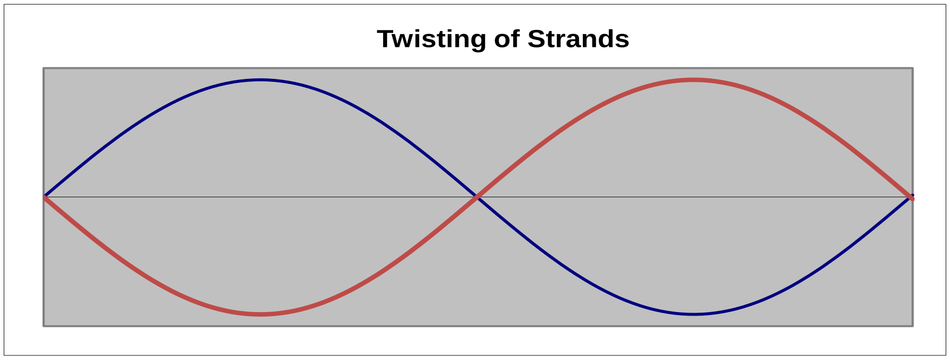

Figure 2.7.1 Twisting of Strands

The strands form a series of loops which link with the alternating magnetic field and this induces an emf in each loop. However for two adjacent loops, such as the two above, the induced emf's are in opposite directions and so cancel, and no circulating current flows. This is true if the intensity of the magnetic field is the same in each loop, but in a coil the intensity changes down the length of the coil, so emf cancelation will not be complete and a circulating current will then flow between strands. To reduce this current the flux difference between loops must be reduced by reducing the length of 'lay' (i.e. more twists).

Welsby in his analysis assumes that the intensity varies linearly from zero for the outer turns of the coil to a maximum value for the inner turns. He then compares the power lost in a twisted cable with that of a cable with no twist and concludes that the amount of twisting required to reduce the power loss by a factor of 10 is when the length of lay is:

$$\ell_t \leq (d_s/d_o)\,\ell \qquad 2.7.1$$

where ℓ_t is the length of lay
ℓ is the length of the cable

So the number of twists should be not less than the ratio (d_o/d_s). Notice that this is not dependent on the actual length of the cable used, so a short length of cable (as would be needed for a small coil) will need as many twists as a longer cable needed for a larger coil. However too many twists will increase the length of the strands and increase the resistance (paragraph 2.6).
Welsby also analyses the loss due to the concentric field set-up by the bundle as a whole and this is similar to the skin effect in a solid conductor. To reduce this loss the strands have to be interwoven, not just twisted, and he concludes that the length of lay of the weaving should also correspond to the above equation. However as noted previously interweaving is only necessary to reduce the loss of a *straight* cable.

Welsby's assumption of linearly varying flux down the coil is open to criticism, because it is known that the major loss in coils occurs at the ends, and Butterworth (ref 6) says ' if the coil is divided into four equal sections the distribution of loss is such that approximately 93% of the loss occurs in the outer two sections'. Since this loss is due mainly to the magnetic fields cutting the wire, Welsby's approximation should perhaps have been to assume a linear intensity reduction over a quarter of the coil length instead of a half length, and this would result in half the length of lay given by Equation 2.7.1. However this point is rather academic given that practical Litz wire has a length of lay much less than that given by Welsby's maximum value above (see Section 6.1).

2.8. Effect of Broken Strands

In a Litz cable with hundreds of strands it is common for some strands to be broken during manufacture, and so measurements have been carried-out by a number of researchers to determine the effect. As expected the dc resistance increases in proportion to the number of broken strands, so that one broken strand in 32 increases the dc resistance by 3.1%. However the ac resistance increases by less than this and Hall (ref 7) found that the ac resistance increased by only about 2% for 32-38 Litz at a frequency of 0.9 MHz. Morecroft (ref 8) also tested the effects of broken strands and an analysis of his data gives ac resistance increasing by about 1.5% per strand for a 48 strand Litz. So in general we can say that the effect of breaking a strand is to increase the ac resistance by K/n compared with the increase in the dc resistance of 1/n, where n is the number of strand, and these two authors put $K \approx 0.65$. However the author's own measurements (see Appendix1) show that K is highly frequency dependent because the current which flows in the broken strands is due to the capacitance between strands and this causes K to vary from unity at low frequencies to zero at high frequencies.

3. PREVIOUS WORK

The most widely quoted evaluation of Litz wire for single layer coils, is by E L Hall (National Bureau Of Standards, 1926, ref 7) who needed to make coils of the highest Q for a standard frequency meter. He investigated coils wound with round copper wire, copper tube, flat aluminium tape and copper Litz, having between 32 and 96 strands. He gave a Table listing the wire types he decided to use in his final equipment as follows:

Frequency Range	Wire
170 - 1022 KHz	96-38 Litz
1 – 2.4 MHz	Solid: No 10 (2.59 mm dia) or No 14 (1.63 mm)
1.3 – 5.4 MHz	0.125 inch copper tube

So he chose not to use Litz above 1022 KHz. However his Litz wire had a strand diameter of 0.1 mm and this is too large for low resistance at frequencies above 1 MHz, and indeed the theoretical analysis given later agrees with him that Litz wire of this diameter is not suitable above this frequency. So Hall *should* have concluded that specifically *96-38 Litz* was not suitable above this frequency, but instead he says :

'At frequencies from 150 to about 1500 KHz the superiority of Litz wire of a large number of strands is shown, but above that limit a large size copper wire or copper tube is preferable'.

To be fair to Hall it is possible that in 1926 the smallest strand available was 0.1 mm diameter and so at the time the statement was reasonable. However, given Hall's statement subsequent workers in the field understandably concluded that Litz was not suitable above about 1.5 MHz and this became accepted wisdom, coming as it did from the respected US Bureau of Standards. So for instance 17 years later Terman (1943 ref 9 p74) states 'Litz wire gives a higher Q than solid wire at the lower radio frequencies, while at very high frequencies solid wire is as good as Litz or better. The frequency at which Litz loses its advantage depends on circumstances, but is normally of the order of 1 or 2 MHz'. As support for this statement he references Hall's paper.

Langford Smith in 1963,(ref 10 p466) states ' Litz wire is most effective at frequencies between 0.3 and 3 MHz. Outside this range comparable results are usually possible with round wire of solid section, because at low frequencies the skin effect steadily disappears while at high frequencies it is large even for the fine strands forming the Litz wire…..'. He gives no reference for this statement.

Wikipedia ('Litz wire') in 2014 gives 2 MHz as the upper frequency, and quotes Terman 1943 as the reference.

4. THEORETICAL MODELS

4.1. Introduction

The author has been unable to find any paper other than Hall's which attempts to make a direct comparison between solid wire and Litz cable. One reason for this is the considerable amount of work required given the large number of variables : coil diameter, coil length, number of turns, conductor diameter to pitch ratio (d_w/p), the number of strands, the diameter of the strands, the insulation thickness, and the frequency. Ideally for a proper evaluation each one of these parameters should be changed one at a time and in combination and the results noted.
It is now possible to carry-out the comparison using computer models, and this is done here, along with sufficient experiments to validate the models.

4.2. Solid Wire Model

The alternating current resistance of single layer coils wound with round wire presents serious mathematical difficulties but was attempted by Butterworth in the 1920's in a series of complicated papers (e.g. ref 6), and summarised by Terman (ref 9). However this analysis has been shown to be not very accurate for the important case of close wound coils (Medhurst ref 11). In 1951 a more accurate theory was developed by Arnold (ref 12), but again the analysis is complicated and the resulting equations difficult to use.
Recently the author has produced an equation which gives very accurate predictions of coil resistance when tested against Medhurst's extensive measurements (Payne ref 1), *and* can be readily programmed into spreadsheets :

$$R_T = R_{ow} [1 + k_r K_n^2 + 16 \pi (1- K_n) (d_w /p)^2_{av} M^2 (l_e / l_c)^2 / (w_2 / w_1)] \qquad 4.2.1$$

where
$R_{ow} \approx 0.25 R_{dc} (d_w/\delta)^2 /(d_w/\delta -1)$ (see Appendix 2)
$R_{dc} = 4\rho l_w / (\pi d^2_w)$ (see Appendix 2)
$1+ k_r = 2\pi [1 /\{ \pi (1+x)\} + 2 (N-1) (1+x) /N (I_{MS} /I^2) / \pi]$
$M \approx d_{coil} /[(2 d_{coil})^2 + (l_{coil})^2]^{0.5}$
$K_n \approx 1/ [1+ 0.45 (d_{coil} / l_{coil}) - 0.005 (d_{coil} / l_{coil})^2]$
$(l_e / l_c) \approx K_n (1+ 0.05 d_{coil} / l_{coil})$
$(w_2 / w_1 = 1/ [1+ 2 (N'-1) / N'(I_{MS} /I^2)]$
$N'=N [1- K_n]$
$(I_{MS} /I^2) \approx 0.0026 – 0.04 d_w/p + 0.404 (d_w/p)^2$
$(d_w/p)_{av} = (d_w/p)$ for N> 8 turns
a_{coil} and l_{coil} are the radius and length of the coil.
d_w is the diameter of the wire
p the pitch of the winding

This equation is valid for coils of any diameter and length, wound with any number of turns N greater than 8, and any conductor diameter to pitch ratio (d_w/p). The only other restriction is that the diameter of the conductor must be greater than 1.6 times the skin depth, $d_w/\delta > 1.6$ (see Appendix 2), so at 1MHz and above the wire diameter should be at least 0.1 mm. This will always be the case here because the comparison with Litz wire is done with a solid wire with a diameter equal to that of the Litz bundle and this will always be more than 0.1 mm for any practical Litz wire with more than 8 strands.

4.3. Litz Cable Model

The resistive loss of a *straight* section of Litz wire is given by Terman (ref 9 p37) as :

$$R_{ac\ straight} = R_{dc} [1+ F + k (n d_s/d_o)^2 *G] \qquad 4.3.1$$

where d_s= strand diameter, d_o= bundle diameter

n is the number of strands in the bundle
$G \approx [d_s/(4\delta)]^4$ accurate to ±5% for $d_w/\delta \leq 4$
F= G/3 see Appendix 2
$k \approx 2 - (1.4/n)$
R_{dc} is for the bundle (ie n strands in parallel)

The factor (1+F) is the resistance increase of an individual strand above its dc value due to its own magnetic field (Appendix 2), and the term $k\,(n\,d_s/d_o)^2{*}G$ is the increase in the bundle resistance due to the magnetic field around the bundle as a whole, the proximity loss.
F, and G are tabulated by Terman (his Table 18), but because the diameter of the strands is assumed here to be small compared with the skin depth, the asymptotic equations above can be used, and these are accurate to within ±5% for $d_s/\delta \leq 4$. Terman p37 also gives a table for k and the above empirical equation provides a fit of ±1.5% for all values of n.

When the Litz wire is wound into a coil there is an additional loss due to the proximity of one bundle of wires to its neighbours, the proximity effect. The overall loss then becomes (seeTerman p80, equation 95, with the terms grouped differently).

$$R_{ac\ coiled} = R_{dc}\,[1 + F + k\,(nd_s/d_o)^2\,G + u(nd_s/p)^2\,G] \qquad 4.3.2$$

The factors F, G and k are as before, and p is the pitch of the winding. The factor u reflects the fact that the proximity loss turn to turn is dependent upon the intensity of the magnetic fields cutting the cable, and these are dependent upon the ratio of coil length to diameter, increasing as this ratio increases. The factor u is the sum of two parts $u_1 + u_2$ due to the radial and axial fields, and these are tabulated by Terman, however the following empirical equation gives the sum of $u_1 + u_2$ to within ±6% for all values of ℓ_{coil}/d_{coil} from zero to 10 :

$$(u_1 + u_2) = 1/[0.18\,e^{-x} + 0.11] \qquad 4.3.3$$

$$\text{where } x = 0.68\,\ell_{coil}/d_{coil}$$

Terman says that Equation 4.3.2 is valid for 'single layer coils of many turns not too closely spaced'. It is not clear from this statement what the limits are for number of turns, N, or the spacing (pitch p), but the equation gave good agreement with measurements of an 84 strand Litz coil with $d_o/p = 0.86$ (see Section 6.6).

5. THEORETICAL EVALUATION OF LITZ

5.1. Comparison Criteria

The above equations show that the loss for both the solid wire and Litz cable is dependent on the winding pitch d_o/p (or d_w/p for solid wire), the ratio of coil length to diameter ℓ_{coil}/d_{coil} the frequency and the number of turns. The Litz cable loss is also dependent upon the strand diameter d_s and the number of strands. In the following all these parameters are changed except the number of turns because the Terman equations for Litz are for 'many turns', and here that is assumed to be 20 turns or more (the more detailed model of solid wire given in Payne (ref 1) shows the resistance reducing by about 4% when the number of turns are reduced from 20 to 5, and so this effect is relatively small).
A fair comparison between solid and Litz would be to choose the diameter of the solid wire over its insulation to be equal to that of the overall cable diameter of the Litz, (which includes *its* overall insulation) since then the same coil could be wound with either Litz or solid with no change to its physical dimensions.
The diameter of the solid wire over *its* insulation is given for small diameter wire by Equation 2.3.2 but for diameters above 0.25 mm, it does not apply. Analysis of manufacturer's data shows that the insulation thickness is between 3.5% and 10 % of the copper diameter. So an average thickness would be 7% but the actual value is not critical. So in the evaluation it is assumed that :

$d_{w\ ins} = d_w\ 1.07$ 5.1.1

where d_w is the diameter of the copper

$d_{w\ ins}$ is the diameter over the enamel insulation

For the comparison with Litz, the diameter of each over its insulation will be equal and so $d_{w\ ins} = d_{cable}$ and so the diameter of the copper in the solid wire, d_w, for a given diameter of cable will be :

$$\mathbf{d_w = d_{cable} / 1.07} \quad \textbf{5.1.2}$$

It is often stated that for solid wire a slightly smaller diameter than that which fills the winding gives a lower loss. The model (Equation 4.2.1) shows that this is indeed true when ℓ_{coil}/ d_{coil} is less than unity, and for instance when $\ell_{coil}/ d_{coil} = 0.4$ a d_w/p ratio of around 0.75 is optimum, although the actual value is not critical. However when $\ell_{coil}/ d_{coil} = 1$ there is very little difference, and for $\ell_{coil}/ d_{coil} \geq 2$ it is best to have as large a diameter of wire as possible. In the modelling which follows the optimum diameter of solid wire was chosen to give the minimum loss, within the constraint that the turns cannot be closer than the enamel insulation allows i.e. from Equation 5.1.2 $d_w/p \geq 1/ 1.07 = 0.93$.

5.2. Typical Results

A typical comparison is given below, for an experimental coil described later.

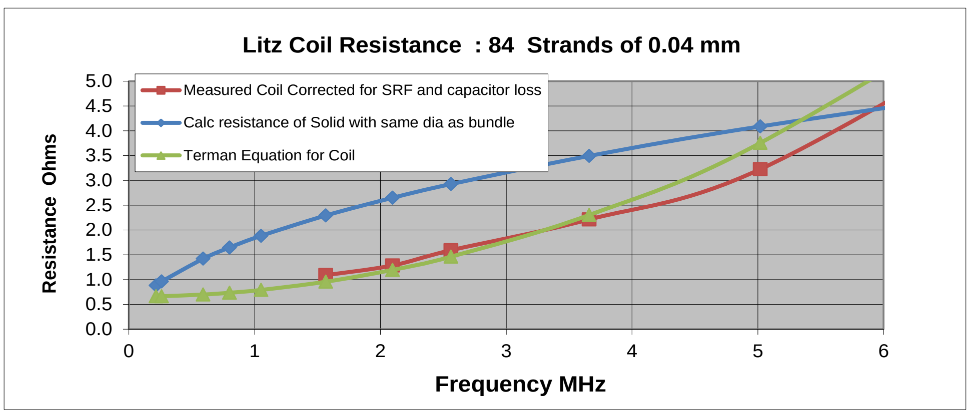

Figure 5.2.1 Comparison of Resistance

It is seen that this Litz coil has lower resistance over a frequency range from 0.2 to 5.5 MHz, when compared with a coil wound with solid wire of the same diameter. The high frequency cross-over occurs when the strand diameter is greater than about 1.5 skin depths, and the low frequency cross-over occurs because the resistance of the solid wire is heading for *its* dc resistance, and this is lower than the Litz because the solid conductor has more copper for the same overall cross-sectional area.

This comparison is quantified by taking the ratio of the resistances as follows :

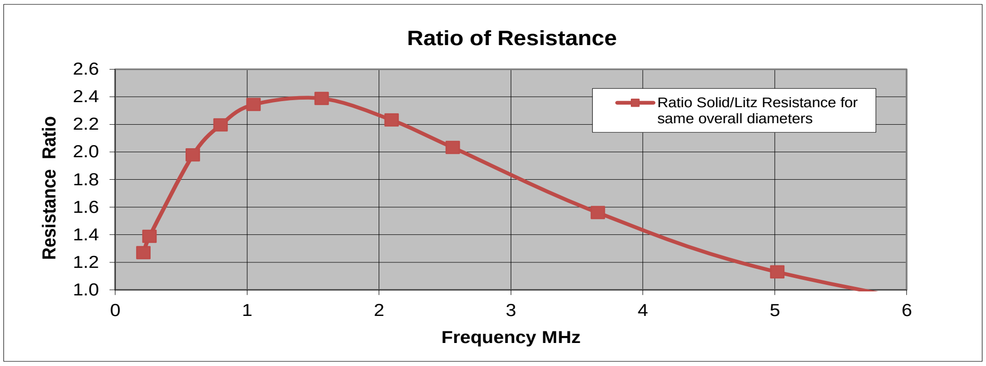

Figure 5.2.2 Resistance Ratio

5.3. Self-Resonance

Equations 4.2.1 and 4.3.2 do not include a term for the rising resistance with frequency due to self-resonance (see Section 6.3). However this increases the resistance of both conductors by the same factor and so the comparison would be unaffected.
In Figure 5.2.1 the measured results (red) have been corrected for SRF (and also for capacitor loss- see Section 6.2).

5.4. Hall's Cable

Using the theoretical models it is interesting to repeat Hall's measurements (ref 7) where he wound 8 turns of 98-38 Litz (strand dia =0.101 mm) onto a grooved former to give ℓ_{coil}/ d_{coil} =0.4 and d_{cable} /p = 0.39.
Assuming a heavy build of strand insulation (Equation 2.3.2) the theoretical model gave the following :

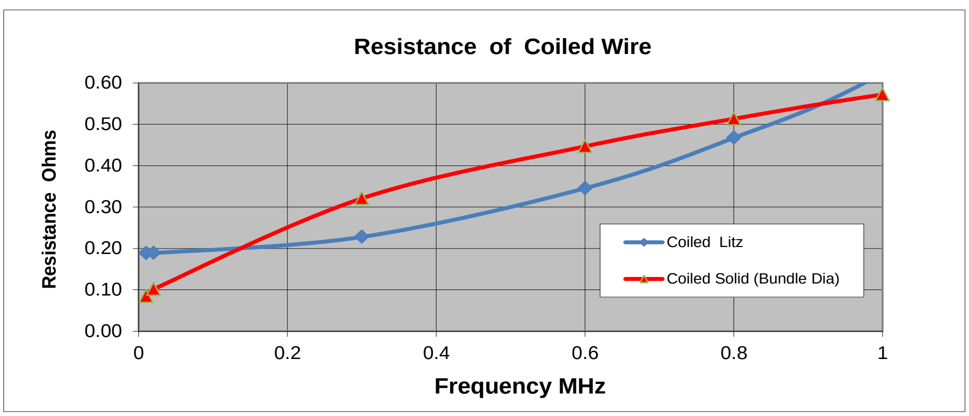

Figure 5.4.1 Resistance Ratio

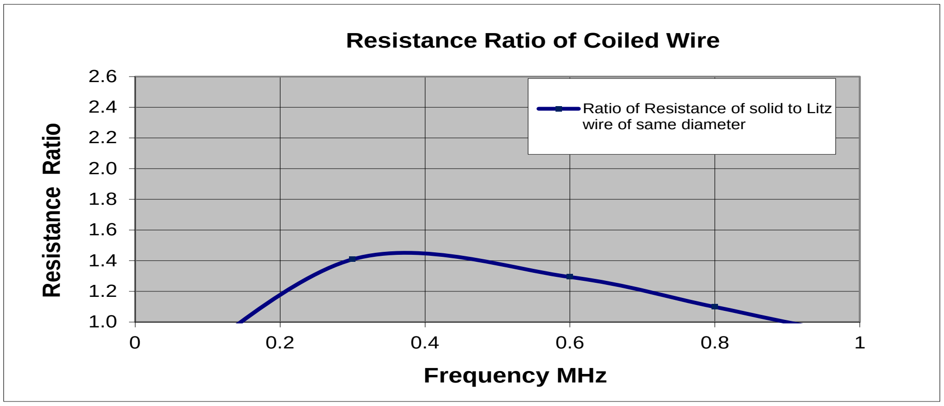

Figure 5.4.2 Resistance Ratio

This shows that this Litz cable has an advantage from 0.15 to 0.9 MHz, although with only 8 turns it is not clear on the accuracy of the Litz model. However some support comes from Hall's decision to use this Litz cable in preference to solid conductors for coils in the frequency range 0.17 to 1.022 MHz.

5.5. Change of d_o /p

The winding ratio d_{cable} /p in the above example was low at 0.39, and if it is changed to a close wound coil with d_{cable} /p =1 (d_o/p = 0.93) the advantage of Litz improves enormously :

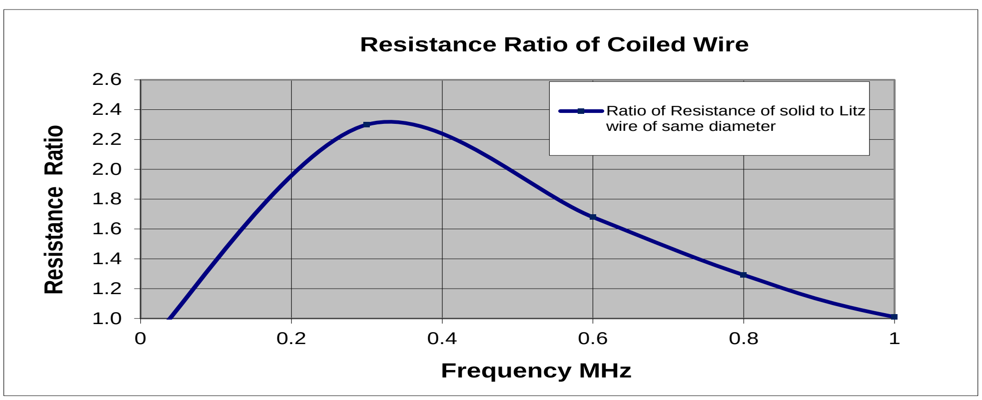

Figure 5.5.1 Resistance Ratio

The above example shows that Litz has its best advantage when the coil is close wound. This advantage was not seen by the earlier researchers because generally their coils were not close wound.
Given that the model shows this large advantage it was important to validate it for the Litz wire when close wound, and this was done with experiments given later. It was not necessary to validate the solid wire equation since this had already been done against Medhurst's experimental results (see Payne ref 1).

Notice that the upper frequencies are not much improved by changing the winding ratio, and this is because this upper frequency is determined mainly by the diameter of the strands, and so this is the next parameter to consider.

5.6. Effect of Strand Diameter

As the strand diameter is reduced the upper frequency increases, because the strand is now small compared with the skin depth to a higher frequency. Also the insulation around each strand is now a larger proportion of the total area and this reduces the ratio d_s/d_o Equation 4.3.2, and with it a reduction in loss. However this larger proportion of insulation reduces the advantage at lower frequencies because of the smaller proportion of copper.

If the strand diameter is reduced to 0.04 mm, and the number of strands increased to 600 to keep the same bundle diameter as the previous examples (1.44 mm) then the model gives (d_o /p = 0.94, d_w /p = 0.84, ℓ_{coil}/ d_{coil} = 0.4, n = 600):

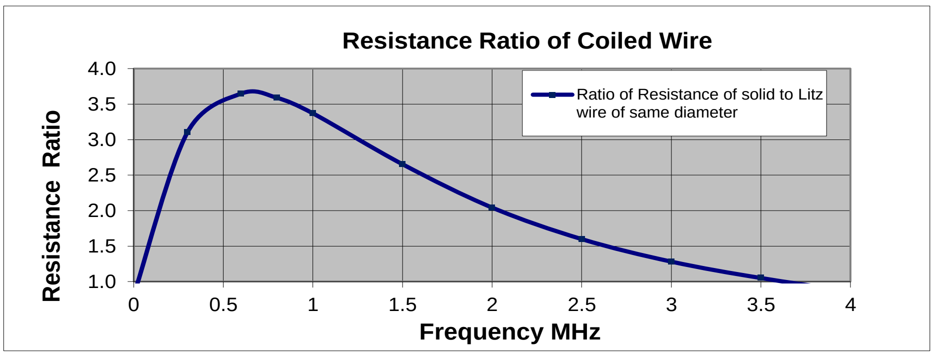

Figure 5.6.1 Resistance Ratio

Notice the change of scale for both axes. The frequency range is extended to 3.5 MHz and the maximum improvement peaks at 3.65.

If the strand diameter is further reduced to 0.025 mm (the smallest readily available) and number of strands held at 600, the model shows a further improvement to 9 MHz (d_o /p = 0.91, d_w /p = 0.84, ℓ_{coil}/ d $_{coil}$ = 0.4, n=600)

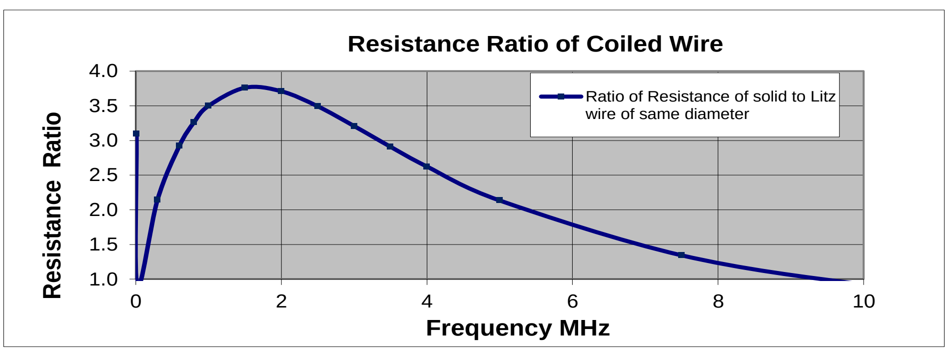

Figure 5.6.2 Resistance Ratio

5.7. Effect of Coil Length/ Diameter

Many coils will have an ℓ_{coil}/ d $_{coil}$ larger than this and for ℓ_{coil}/ d $_{coil}$ = 2 the model gives (d_s = 0.025 mm, d_o /p = 0.91, d_w /p = 0.93, n =600)

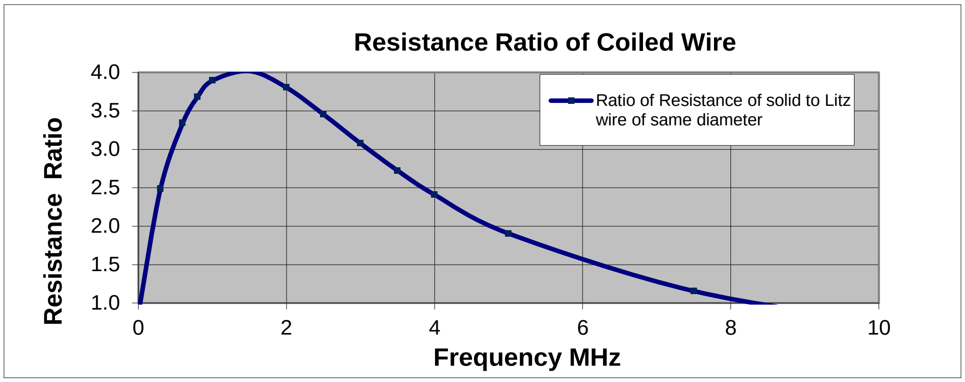

Figure 5.7.1 Resistance Ratio

The resistance ratio is not too dependent on the ratio ℓ_{coil}/ d $_{coil}$, as shown by the small change in the above curve from that of Figure 5.6.2 (ℓ_{coil}/ d $_{coil}$ = 0.4 and 2 respectively).

5.8. Effect of Number of Strands

If the parameters in Para 5.7 are used, but with n reduced to 126 (an available cable), the model gives (for ℓ_{coil}/ d $_{coil}$ = 2, d_s = 0.025 mm, d_o /p = 0.85, d_w /p = 0.88, n =126) :

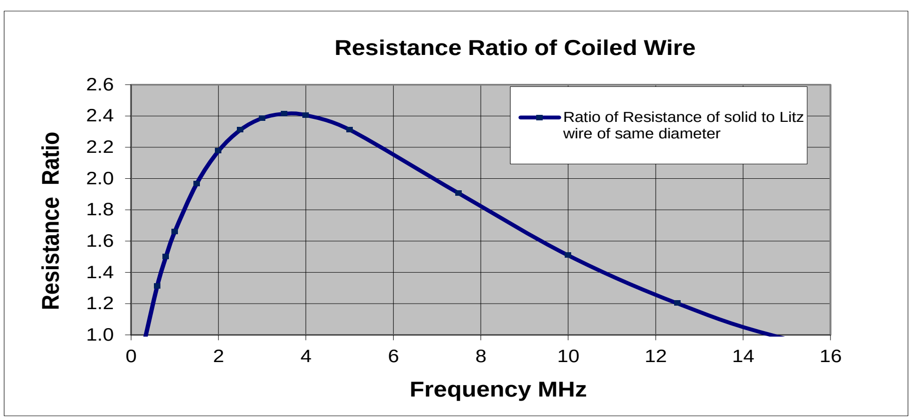

Figure 5.8.1 Resistance Ratio

Although d_{cable} / p was set at unity in the above, this corresponded to d_o /p of only 0.85 because the overall silk insulation is proportionately larger with this smaller bundle diameter.

Comparing the above figure with Figure 5.7.1 shows a reduction in the value of the peak improvement. One reason is that when the number of strands is small the bundle diameter is also small, and so the proportional area taken by the overall silk insulation increases. However the main reason for the reduced peak can be seen by considering just the solid conductor, which now has a smaller diameter because the bundle diameter has reduced. At a given frequency the conducting area of a *small* diameter solid wire is a greater proportion of its total area than that of a large diameter wire, since the skin depth is the same in both cases. So the solid wire is more effective when its diameter is small, and the extreme example of this is when its diameter is so small that it is less than the skin depth when the whole of the cross-section carries current. This advantage of the smaller solid diameter is reduced at high frequencies because the skin depth is smaller and then the Litz gives the smaller resistance.

5.9. What if ?

The above theoretical experiments have used values for strand diameter and insulation thickness of commonly available Litz cable. However it is interesting to use the model to see the performance which might be achieved if there was freedom to choose the most advantageous values. Assuming that the objective is to improve the high frequency performance then this will of necessity require a smaller strand diameter, and the smallest which is being offered is 0.016 mm (ref 13), albeit as a special and on request. Assuming the maximum number of strands is 270 (as per the standard products offered by this company), and an insulation build equal to the average of Equation 2.3.2, the model gives (for $\ell_{coil}/d_{coil} = 2$, d_s = 0.016 mm, n =270, $d_o/p = 0.81$, $d_w/p = 0.93$) :

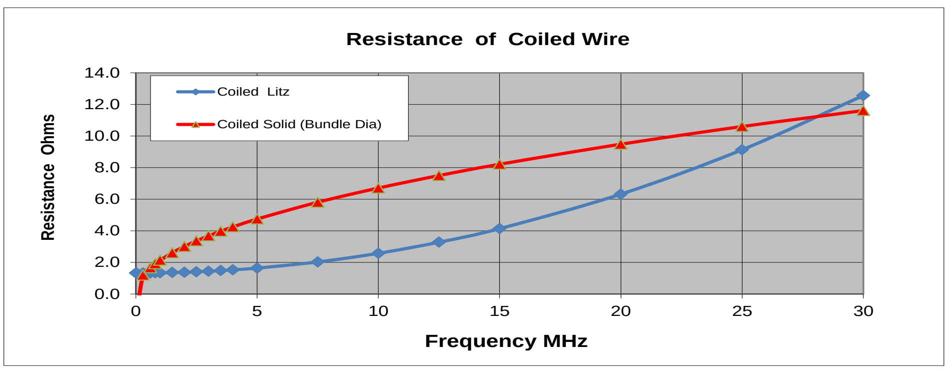

Figure 5.9.1 Resistance Ratio

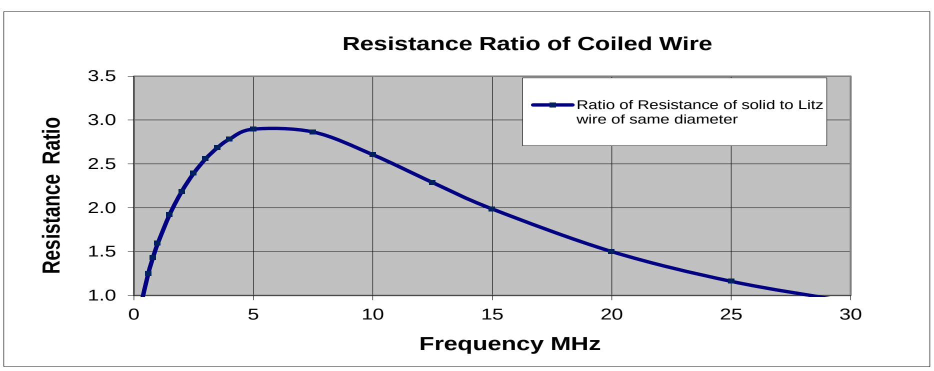

Figure 5.9.2 Resistance Ratio

This cable would therefore provide a useful improvement over solid wire up to at least 20 MHz.
The bundle diameter and wire diameter were 0.45 mm for the above, but at high frequencies only a small number of turns are necessary to achieve the required reactance, and so larger diameter cable can be used. This could be achieved with a larger number of strands but then the high frequency performance reduces because of the increase in the bundle proximity loss. The alternative is to increase the thickness of insulation around each strand, and if this is increased to twice the heavy build thickness (Equation 2.3.2) with all other parameters unchanged this gives :

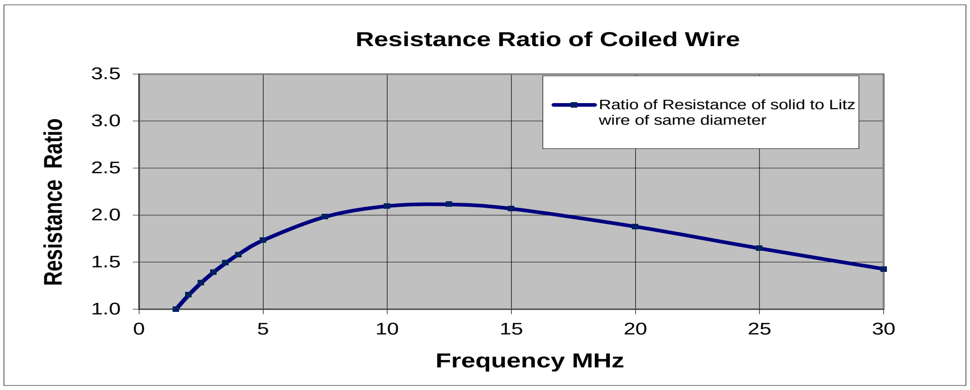

Figure 5.9.3 Resistance Ratio

The bundle diameter and wire diameter are now 0.87mm. The performance at high frequencies has improved, but the low frequencies have degraded because of the lower copper content of the Litz.

6. MEASUREMENTS TO SUPPORT THEORY

6.1. Intro

Several coils were wound with Litz cable and their resistance measured, using an Array Solutions vector network analyser. The impedance of the test jig was calibrated-out using the analyser's internal calibration facility.

6.2. Tuning Capacitor

The reactance of the coils was much higher than their resistance and this led to severe inaccuracies in the measurement of resistance. So for all measurements the inductive reactance was tuned-out with variable air capacitors, whose resistance was assumed have the form :

$$R_c = R_s + \alpha/(f\, C^2) + \beta\, f^{\,0.5} \qquad 6.2.1$$

The parameters R_s, α and β were determined by previous measurements (see Payne ref 14). This resistance was subtracted from all loss measurements.
The value of the capacitance C at each frequency was calculated from the inductance L, using
$f = 1/\,[2\pi(LC)^{0.5}]$. However L varies with frequency due to self-resonance (see Section 6.3), and so was measured at each frequency.

The estimated error in the calculation of the capacitor series resistance was ±13%, but at the minimum capacitance setting where the vanes are not meshed, the error is probably much higher.

6.3. Self- Resonance

The measured resistance and inductance increase with frequency due to self-resonance, and a good estimate of this increase is given by the following equations (see Payne ref 15) :

$$L = L_m \left[1- (f / f_r)^2 \right] \qquad 6.3.1$$

$$R = R_m \left[1- (f / f_r)^2 \right]^2 \qquad 6.3.2$$

L and R are the values which would obtain in the absence of self-resonance, and L_m and R_m are the values which would be measured at any frequency.

The above equations are accurate when the measurement frequency is low compared with the SRF (say $f < 0.3 f_r$), but become increasingly inaccurate at higher frequencies. It is important therefore for the SRF to be as high as possible, and for the measurement jig to have minimal effect. In particular, the series tuning capacitor gives a parallel capacitance merely due to its volume, and so its body should be grounded and not placed at the high potential end of the inductor. As an example in an experiment the SRF was reduced from 21.5 MHz to 14 MHz by placing the capacitor at the ‘hot’ end of the coil.

6.4. Winding the Coils

When a close wound coil was required, i.e. a d_{cable}/p as close as possible to unity, it was important that each new turn was pressed against the previous turn as it was wound on, and this was most conveniently done with the thumb. To avoid flattening of the cable against the winding former low tension was applied. The winding was fixed to the former with Sellotape at each end.

Soldering the cable ends was relatively easy in that the strand insulation was ‘solderable’, in that it was removed by the heat of the soldering. However it was important to ensure that every strand was tinned, and this was achieved by fanning out the strands onto a wooden block and brushing the soldering iron tip across them while applying cored solder. Once all strands were tinned they tended to clump together due to surface tension.

The coils tested are shown in Figure 6.4.1, along with a mm scale :

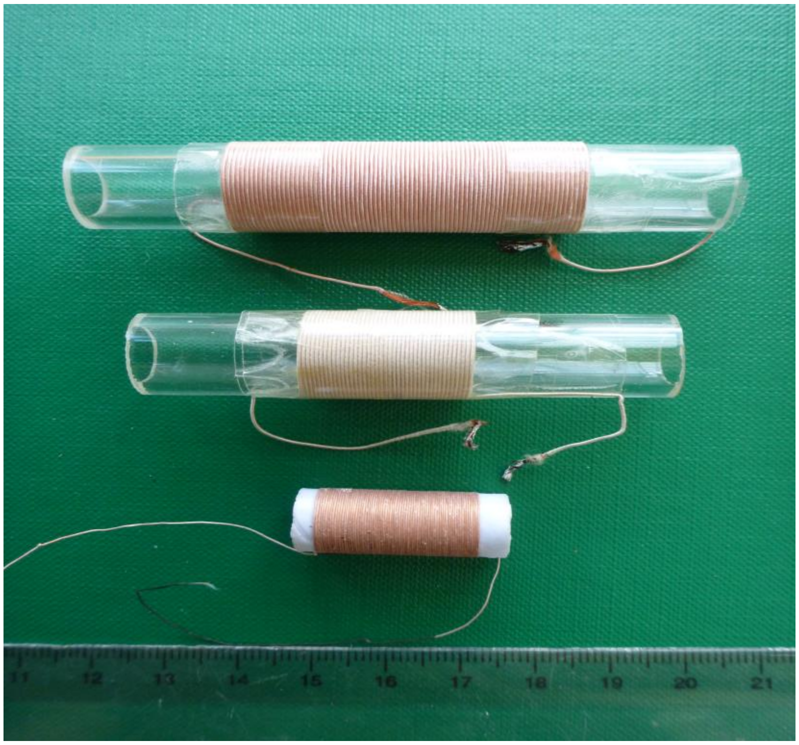

Figure 6.4.1 The coils tested

It was important to determine the parameters of the Litz cable being tested and this posed a number of problems as follows.

Strand diameter is difficult to measure, firstly because the strands are so small and secondly because they are covered with insulation. One option is to deduce the diameter from a measurement of the dc resistance of an individual strand, and indeed at least one manufacturer recommends this approach for small strands. However this will be in error for the cable as whole if the other strands have a slightly different diameter. In a cable with 100's of strands it is not feasible to measure the dc resistance of all the strands individually and so the resistance of whole cable can be measured and multiplied by the number of strands. However this will also be in error if there are any broken strands. Also the whole cable will likely have a resistance below 1Ω and the measurement of this low value will be subject to larger errors than that of the strand resistance. Overall, the measurement of an individual strand is likely to be the most accurate measure, but in the following it is the cable dc resistance which was measured and from this the strand diameter calculated.

The diameter of the cable d_{cable} could be determined with a micrometer or vernier, but the pressure which these instruments apply distorts the cable resulting in a large potential error. The method used here was to close-wind 10 turns or more and measure the length of coil, and divide by the number of turns (see also Section 6.4).

The thickness of the strand insulation and that of the overall ('silk') insulation are important to the performance of the cable, but are very difficult to measure, and the author has not found a satisfactory method. However if it is assumed that the silk insulation is 0.04 mm thick (Section 2.4) then the bundle diameter d_o can be determined from the cable diameter above, and from this and the strand diameter the insulation thickness can be calculated.

The length of lay of a cable is not easy to measure because the outer insulation has to be removed and this can distort the underlying bundle. Fortunately the length of lay does not need to be measured accurately because the standard lay is much less than required to minimise losses (Section 2.7). As an example, the 126/0.025 cable used later had a measured length of lay of around 13 mm although the error on this measurement is probably ± 3 mm. So with the cable length of 3.9 meters used to wind the coil described later this gave the number of twists at between 240 and 390 twists, so easily exceeding Welsby's minimum value according to Equation 2.7.1 of 16 twists.

6.5. Measurements of True Litz 11/0.042

Litz cable was unwound from a second-hand ferrite antenna, and inspection showed that the cable was both twisted and woven. This had 11 strands, and the strand diameter was determined as 0.042 from the dc resistance. The bundle diameter d_o was determined by arranging 11 coins in the most dense packing possible (11 circles do not pack well) and the bundle diameter was found to be about 4 d_s= 0.2 mm, and 0.18 mm gave better agreement with theory for the straight wire below. The length was 3.2 M.
Initially the resistance of the straight (uncoiled) cable was measured. Of course it is not possible to measure a truly straight cable because the ends have to be close together at the measuring equipment, but it was found that folding the cable in half so that it was similar to a shorted two wire transmission line, and with a spacing of about 20 mm between the two halves, the resistance was sensibly equal to that of a straight cable.
So for n=11, d_s = 0.042 and d_o = 0.18 mm Equation 4.3.1gave the following in brown (note the offset y axis) :

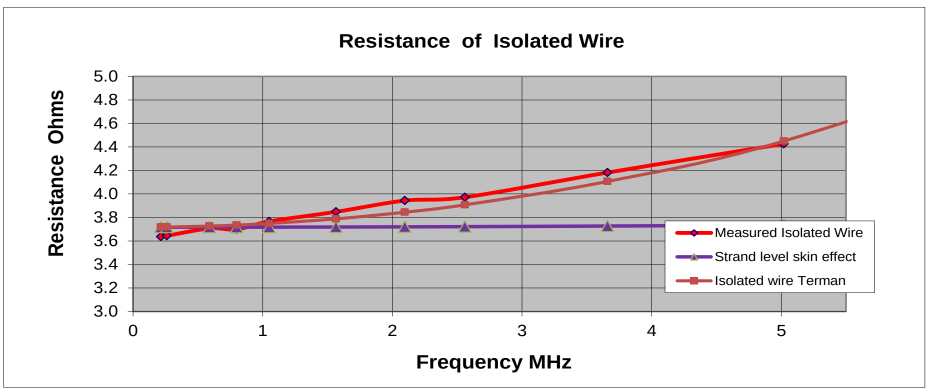

Figure 6.5.1 Resistance Ratio

In red are the measured values, and the agreement is within 2.5%. Also shown is the calculated strand resistance (purple) R_{dc} (1+F) and this increases only 2% over this frequency range, so the increase in the resistance of the straight cable is predominantly due to the proximity loss R_{dc} (k (n $d_s/d_o)^2$*G
This cable was wound onto a thin plastic tube (0.5 mm thick and likely to have been polystyrene) to give about 90 turns, with a mean diameter of 11.35 mm over a length of 25 mm, so pitch was 0.278 (see bottom coil of Figure 6.4.1). So this coil was not tightly wound (d_o/p=0.65), and thus should agree well with Equation 3.3.2
The inductive reactance was tuned-out with a variable air capacitor and the loss of this capacitor subtracted from the measured resistance as per Section 6.2. The measured resistance values were also corrected for SRF (Section 6.3) and also a small calibration error, to give the following (in red):

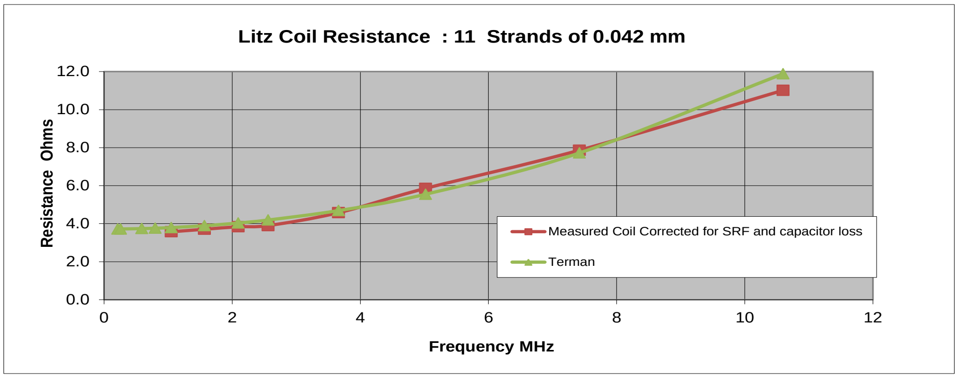

Figure 6.5.2 Resistance of coil

Also shown in light green is the calculated resistance from Equation 4.3.2. The agreement is generally within ±5% over the whole range except at the highest frequency where the error increases to 8%. The error here is likely to be in the measurements, because at this frequency the vanes in the tuning capacitor were almost unmeshed and the equation for predicting the capacitor loss could then be in error (see Section 6.2).

So this experiment confirmed Terman's equation, at least for a small number of strands, and with a loose winding ratio ($d_o/p=0.65$).

6.6. Measurements of Litz 84/ 0.04

A cable was purchased having 84 strands of 0.04 mm diameter This cable was claimed by the supplier (ref 16) to be Litz but inspection under a microscope showed that the bundle had been twisted as a whole but no indication that it had been woven. All strands were insulated with enamel and the whole insulated with silk.
The dc resistance was calculated as 0.637 Ω for the length of 4 metres, and measured as 0.637 Ω. The measurement accuracy was ± 1% so a broken strand was unlikely, since this would have increased the resistance by 1.2%. Given the measurement uncertainty the strand diameter could have been 1% smaller at 0.0396 mm, and this indeed gave a better correlation with the ac measurements (resistivity was assumed to be $1.68\ 10^{-8}$ ohm m).
The measured and predicted resistance are shown below, assuming an average insulation build (Equation 2.3.3) :

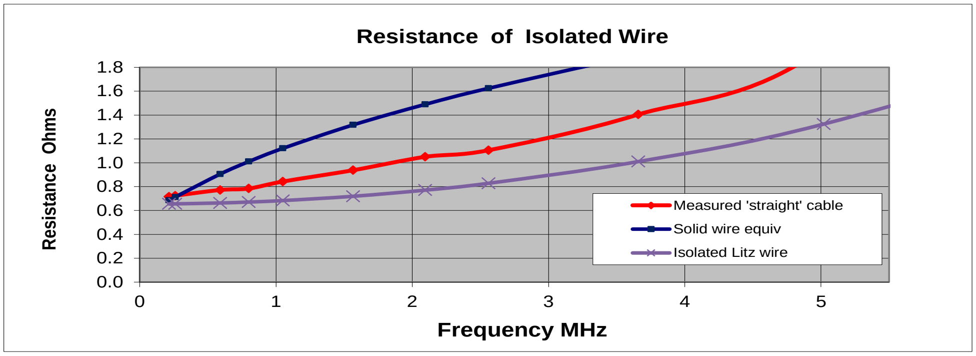

Figure 6.6.1 Resistance of 'Straight' Cable

The measured resistance was corrected for an SRF of 22 MHz (measured) and the resultant shown in red. The resistance is much higher than predicted (purple), but not as high as a solid conductor with the same area of copper (blue). This suggests that the cable had some weaving but not sufficient to minimise the resistance (see Section 2.2).
The resistance of the chosen length is small and this leads to measurement errors including an increased relative error for the uncertainty in the capacitor resistance. It would have been preferable therefore to use a longer cable, but the SRF when wound into a coil would then have been close to the maximum frequency of interest (10 MHz), and the correction for SRF would then be inaccurate.
This wire was wound onto a tubular former, leaving 50 +50 mm for leads, so the wound length was 3.9 m. The former was of clear acrylic having an outside diameter of 14.95 mm and a wall thickness of 2.2 mm. The wire was close wound with exactly 80 turns, over a length of 45 mm to give $d_o/p = 0.85$ and a mean winding diameter of 15.5 mm (top coil of Figure 6.4.1) The following resistance was measured (red) :

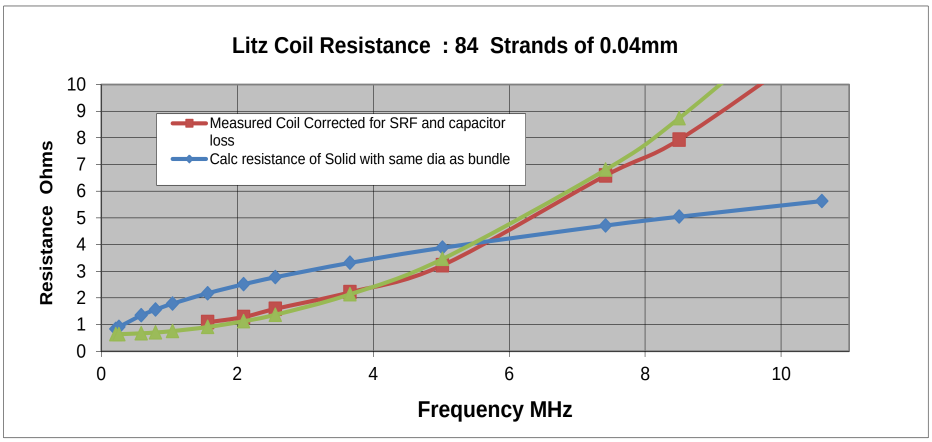

Figure 6.6.2 Comparison of Theory and Measurements

The measurements follow closely the prediction from the Terman equations (green), up to 7.5 MHz. Above this frequency the corrections for SRF and capacitor resistance become a significant proportion of the total resistance and so contribute to a large uncertainty.
Also shown is the predicted resistance of solid wire having the same diameter as the Litz bundle (blue), and this is has a higher resistance from 0.2 MHz to 6MHz.
Given that the resistance of this cable when straight did not comply with Terman's equation, but when coiled did, this supports Welsby's contention that for a *coil* with more than 10 turns the strands need be twisted only and not woven and that weaving is only necessary to reduce the resistance of a *straight* cable.

6.7. Measurements of Litz 126/0.025

A cable was purchased having 126 strands of 0.025 mm diameter (ref 16), and this is the smallest strand diameter readily available. This cable was claimed by the supplier to be Litz but inspection under a microscope showed that the bundle had been twisted as a whole but no indication that it had been woven. All strands were insulated with enamel and the whole insulated with silk with an overall diameter over the silk of 0.506 mm as measured by winding 41 turns closely and measuring the overall length of 20.7 mm. This is consistent with Equations 2.3.1, 2.3.2, 2.3.3 and 2.4.1, if *heavy* build insulation is assumed. The coil used a 2 metre length close wound onto an acrylic tube of 14.98 mm OD and 10.49 mm ID (middle coil of Figure 6.4.1).
The dc resistance of a 2.1 m length was calculated as 0.570 Ω and measured as 0.602 Ω, 5.6% higher. This could be explained if 7 strands were broken but this seems unlikely over this short length. Alternatively the strand diameter could be slightly smaller than 0.025 mm and indeed better correlation with the ac measurements was achieved with 0.024 mm (4% smaller) (resistivity was assume to be $1.68\ 10^{-8}$ ohm m, and measurement uncertainty was ±1.1 %).

The model predicted that this cable would give lower resistance than a solid conductor up to around 15 MHz, and this was confirmed by the following measurements :

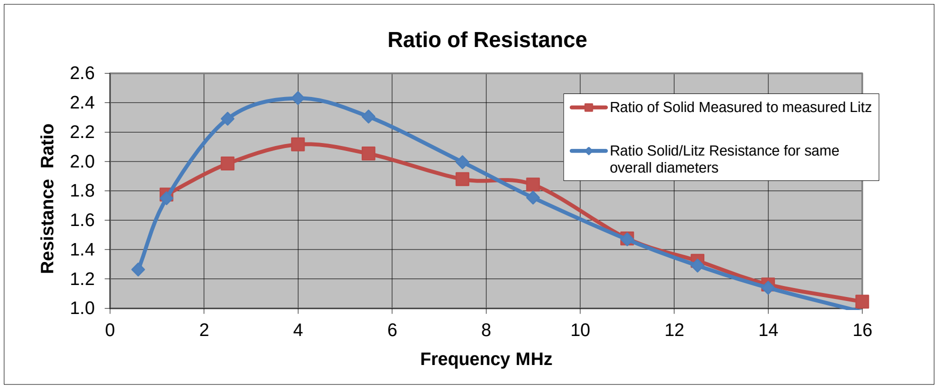

Figure 6.7.1 Resistance ratio

Correlation is very good at high frequencies and provides convincing evidence of the applicability of the model to these high frequencies and close winding. The error is greater at the lower frequencies because here the resistance was less than 1.5 Ω, and the measurement equipment was not very accurate at this level (probably ±12%) and the correction for the (uncertain) capacitor resistance a larger proportion of the total.

6.8. Litz Close Wound

The coils described above in paragraphs 6.6 and 6.7 were as close wound as possible but the d_o/p ratio was somewhat less than unity at 0.86 because of the thickness of the overall bundle insulation (see paragraph 2.4). The measured ac resistance of these two coils correlates well with Terman's equation 4.3.2 (along with 4.3.3) and so it can be assumed that $d_o/p = 0.86$ is within his definition of 'not too closely spaced' (see paragraph 4.3).

6.9. Un-insulated Strands

This is work in progress.

7. TESTING STRANDED CABLE

The following is from Welsby (ref 5) :

'It has been pointed out that a stranded cable must be correctly made up if the full advantage of the stranding are to be obtained. The question immediately arises of finding a method of testing a sample of stranded cable, to determine whether or not it has been adequately twisted in order to make the losses caused by circulating currents negligible. One way is to make a "twin wound" air-cored solenoid : ie a coil in which the cable is wound double so that when completed, the coil consists of two separate, closely spaced windings. The resistance of one winding of the coil is now measured at a frequency high enough to ensure that most of the losses are due to eddy-currents, precautions being taken to see that the cable insulation is kept dry to minimize dielectric loss. First a measurement is made with the other winding open-circuited, but with all the strands bonded together at each of the two free ends. Then, without changing the measuring conditions in any way, the bonded ends of the second winding are snipped off, and the insulated strands fanned-out, so that they cannot come into contact with each other. If the cable has been adequately twisted, there will be no change in the resistance measurement. A drop in measured resistance, however, will indicate that the losses caused by circulation currents, when the ends were bonded, were not negligible'.

This test was carried-out by the author on the 126/0.025 cable at 4 MHz. The resistance decreased when the ends were snipped-off but by only 0.65%, and so this cable was adequately twisted.

Welsby goes on to say 'This test deals with the effect of the perpendicular field only. A check of the circulating currents caused by the internal field of the cable can be made by comparing the ac resistance of a straight length of the cable with the figure obtained by measuring the average ac resistance of one strand , and then dividing this by the number of strands. The two results should agree, within the limits of experimental error, but, if the circulating currents are causing trouble, the resistance of the complete cable will be greater than that of the individual strands in parallel'.

8. DISCUSSION AND SUMMARY

It is shown that using Litz cable readily available on the internet has lower resistance than solid wire of the same overall diameter at frequencies up to 15 MHz. The key parameter in such a cable is the strand diameter and this largely determines the upper frequency range. As a guide, the cross-over frequency where Litz loses its advantage over solid wire is as follows :

Strand Dia (mm)	Cross-over (MHz)
0.1	0.95
0.07	1.9
0.04	5.8
0.025	15

The above is for 100 strands, and the corresponding bundle diameters are 1.46, 1.06, 0.66, and 0.45 mm respectively. This number of strands is approximately the optimum number for best high frequency performance and this is illustrated by the following : for the 0.025 mm diameter strands the diameter of the overall cable is around 0.45 mm. A cable with twice this diameter (0.9 mm) would require 490 strands, but the cross-over frequency is then reduced to 9 MHz, because of the bundle level skin effect. However this is still an excellent performance.

If the number of strands is reduced below 100, the cross-over frequency increases, but only slightly and at the expense of much poorer performance at lower frequencies. This is because the bundle diameter reduces and with it the diameter of the solid wire equivalent and this is more efficient because the skin depth becomes a greater proportion of *its* diameter.

9. APPENDIX 1 : EFFECT OF BROKEN STRANDS

When a strand is broken the dc resistance increases by 1/n where n is the number of strands, but the ac resistance increases by a smaller amount K_s/n where K_s is less than 1. A number of researchers have measured this and typically find that $K_s \approx 0.65$, however none has provided an explanation for this effect.
To investigate this, the resistance of the coil described earlier wound with 85/0.042 Litz was measured, strands removed and the resistance re-measured. Ideally this would be done one strand at a time, but the change in resistance was then too low to measure with sufficient accuracy. So 9 strands were separated off and soldered together at their end. The coil resistance was then measured both with and without these strands connected, and the ratio used to determine K_s as follows:

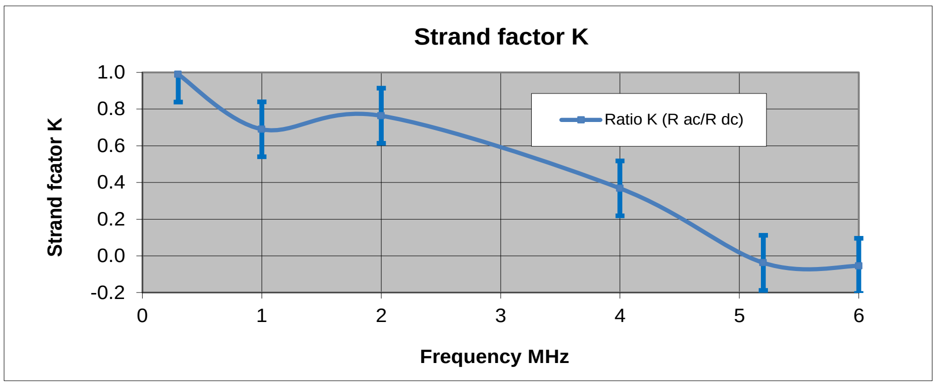

Figure 9.1 Effect of broken Strands

It was very difficult to measure the change in resistance with any accuracy and so the error bars are quite large. Nevertheless a clear trend is evident with the strand factor K_s reducing with frequency from around unity (ie ac resistance reduces as the dc resistance) to zero (no change in resistance with strands disconnected). At the lower frequencies there is a trend to around K_s =0.7, similar to the findings of Hall and Morecroft.
The reduction with frequency suggests that capacitive coupling between strands is causing current to flow in the disconnected strands. This capacitance was determined by isolating one strand from the and measuring the impedance between this strand and all the others as a bundle. This gave the following :

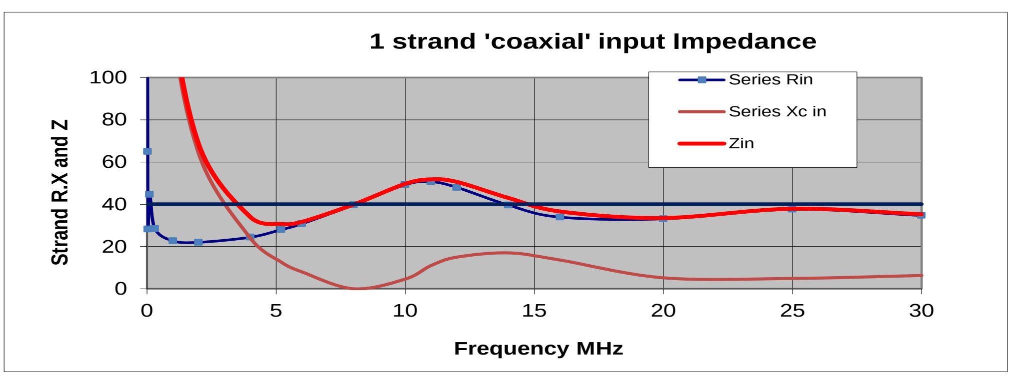

Figure 9.2 Single strand impedance

The series input impedance (red) is mainly capacitive up to 4 MHz and the reducing reactance between strands explains the previous curve. The input impedance is essentially resistive at all higher frequencies. Also shown is the resistance of a single strand, which is essentially constant over this frequency range at around 40Ω.
The capacitance was about 1152 pf for this 4 meter length of cable (ie 288 pf per metre) and its reactance exactly matches the input impedance up to 5 MHz.
For a capacitive current to flow there must be an electric field between the broken strand and the other strands. Of course such a field exists because the broken strand is held at the potential of its connected end whereas the potential of the connected strands increases down their length to a maximum at the other end. In contrast an *unbroken* strand has the same potential along its length as all the other strands, and so no displacement current will flow.
While this is true for the electric field due to the potential between the cable ends, there is also an electric field generated by the transverse magnetic field across each loop (Figure 2.7.1). This will cause circulating currents in the cable as explained in Section 2.7, but it will also produce displacement currents in the dielectric between the strands, and these will give a power loss and an apparent increase in resistance of each strand. However there is no indication in the experimental results that this effect is significant and so it has not been persued.

10. APPENDIX 2 : Skin Effect

10.1. Skin Depth δ

In the case of a wide flat conductor, the current which is set-up on the surface penetrates into the surface exponentially according to the resistivity of the material, its permeability and the frequency. For a current density J_o at the surface, the density at depth z is given by (see Wheeler ref 17) :

$$J_z = J_o\, e^{-z/\delta} \qquad 10.1.1$$

where $\delta = [\rho/(\pi f \mu)]^{0.5}$
$\mu = \mu_r\, \mu o$
μ_r is the material relative permeability
$\mu o = 4\pi\, 10^{-7}$
ρ = resistivity (ohm-metres)

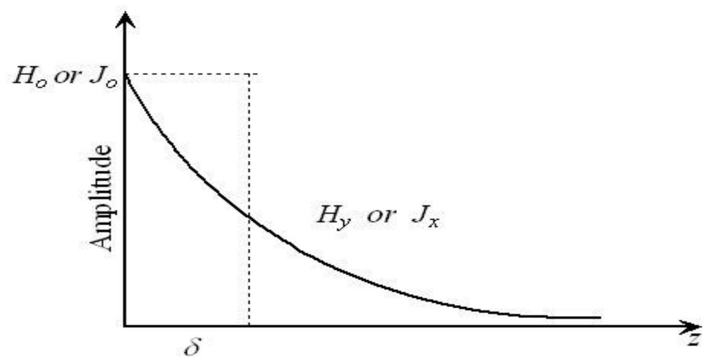

Figure 10.1.1 Current density at high frequencies

Thus the current density decays exponentially as shown in the above curve, and so the *total* current is equal to the area under this curve, integrated to infinity. The area under the dotted curve is equal to this and the depth δ in the above figure is known as the skin depth and defined as follows:

$$\textbf{Skin depth } \delta = [\rho/(\pi f \mu)]^{0.5} \qquad \textbf{10.1.2}$$

So the total power loss associated with this exponential decay of current is the same as if the current was uniformly distributed down to depth of δ, and zero at greater depths. This is a very useful representation for the calculation of the resistance of conductors at high frequencies, as shown in the next section.

The skin depth in mm for various metals is given below (Welsby ref 5). The frequency f is in Hz.

Copper	:	$66.6 / \sqrt{f}$
Aluminium	:	$83 / \sqrt{f}$
Brass	:	$127 / \sqrt{f}$
Nichrome resistance wire	:	$500 / \sqrt{f}$
Mu metal	:	$4 / \sqrt{f}$

The skin depth in copper for frequencies from 1 KHz to 1000 MHz is shown below.

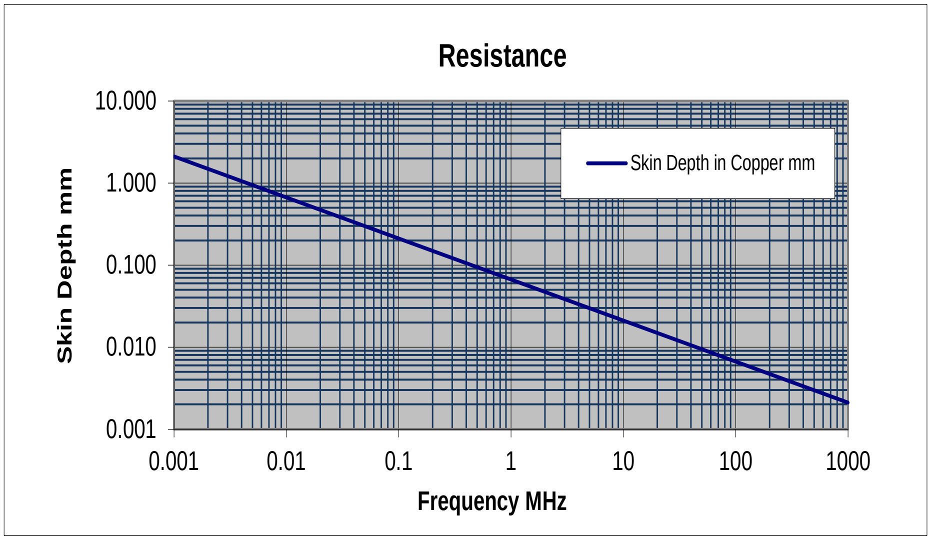

Figure 10.1.2 Skin depth in copper

10.2. Skin Effect in Round Wire

Equation 2.2.1 is for a flat conductor and a circular conductor has been analysed by Ramo & Whinnery (ref 18 p242). They show that the distribution of current involves complex Bessel functions Ber and Bei (i.e. real and imaginary) :

$$i_z = i_o \,(\mathrm{Ber}\,\sqrt{2}\, r/\delta + j\,\mathrm{Ber}\,\sqrt{2}\, r/\delta) \,/\, (\mathrm{Ber}\,\sqrt{2}\, r_o/\delta + j\,\mathrm{Ber}\,\sqrt{2}\, r_o /\delta \qquad 10.2.1$$

where r_o is the radius of the conductor and r is the radius of the field

The solution to this equation is complicated but if the skin-depth is small compared to the conductor diameter so that the effect of the curvature is small, then an exponential decay as Equation 10.1.1 is a good approximation. In that case it can be assumed that the current flows in a hollow cylindrical shell having the same outside diameter as the wire and having a thickness δ :

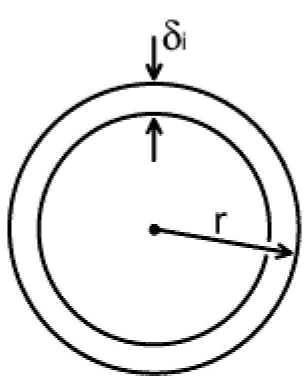

Figure 10.2.1 Skin depth at high frequencies

The resistance of such a tube will be equal to $R = \rho\,\ell/A$, where A is the area of the cross-section. So for a wire of outside radius r_w :

$$R_{hf} \approx \rho\,\ell\,/[\ \{\pi\, r^2_w\} - \{\pi\,(r_w - \delta)^2\}]$$

$$= \rho\,\ell\,/[\pi\,(d_w\,\delta - \delta^2)] \qquad 10.2.2$$

It is useful to express this HF resistance in terms of the direct current resistance $R_{dc} = \rho\,\ell/A = 4\,\rho\,\ell/(\pi\, d^2_w)$. So we have :

$$\mathbf{R_{hf} / R_{dc} \approx 0.25\ \ d^2_w\,/(d_w\,\delta - \delta^2)]} \qquad \mathbf{10.2.3}$$

Dividing top and bottom by δ^2 puts this equation in terms of d_w/δ, which is sometimes more useful:

$$\mathbf{R_{hf} / R_{dc} \approx 0.25\ \ (d_w / \delta)^2\ /(d_w/\delta - 1)]} \qquad \mathbf{10.2.4}$$

This equation is plotted below in green, along with the accurate values of Equation 10.2.1 as tabulated by Terman (ref 9) in blue.

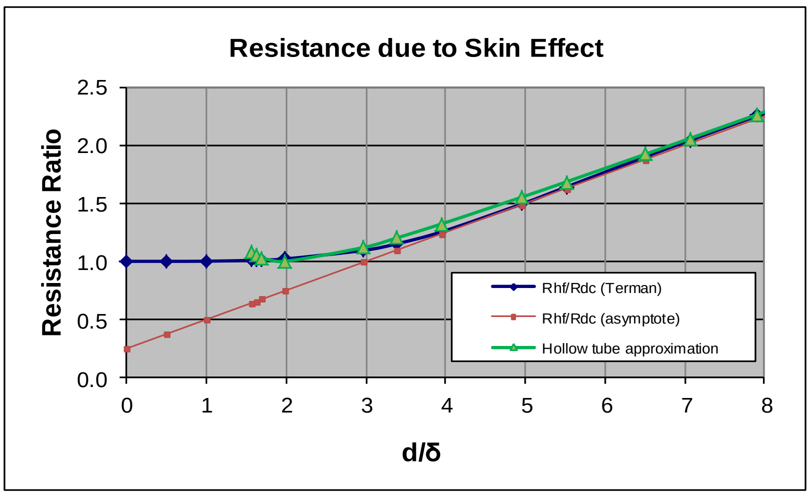

Figure 10.2.2 Resistance at high frequecies

The above equations agree with the tabulated values to within ±5.5% for all values of $d_w/\delta \geq 1.6$. So these equations are surprisingly accurate given the simplification.

Also shown is the asymptote of Equation 10.2.1 for large values of d_w/δ, and this is given by :

$$R_{hf} / R_{dc} \approx (d_w/\delta + 1)/4 \qquad 10.2.5$$

The error is less than -1.3% for all values of d_w/δ above 4. This equation is sometimes approximated to $R_{hf} \approx 0.25\, R_{dc}\, d_w/\delta$ but this is much less accurate for the values of d_w/δ encountered in HF work.
For small values of d_w/δ, Ramo & Whinnery give the following approximation :

$$\mathbf{R_{hf} / R_{dc} \approx 1 + [\ (d_w/\delta)^4 / 768]}$$

$$\mathbf{= 1 + [d_w/(4\delta)]^4 / 3} \qquad \mathbf{10.2.6}$$

This is accurate to within 5% for $d_w/\delta \le 4$. Notice that this equation is the same as the value of (1+F) in Equation 4.3.1.

So the solution to Equation 10.2.1 can be approximated for large wire diameters by Equation 10.2.4, and for small wire diameters by Equation 10.2.6, where the changeover between the two equations is when the value of d_w/δ is 2.8 ±1.2.

NB a number of references quote a formula from the Bureau of Standards for small values of d_w/δ as $R_{hf} / R_{dc} \approx [(1+ q^4 / 48)^{0.5}]/2$ for $q<3$, where $q= (\sqrt{2}\, ro/\delta)$. However this equation can be shown to be the same as Equation 10.2.6 for values of d_w/δ such that $(1+x)^n \approx 1+nx$.

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